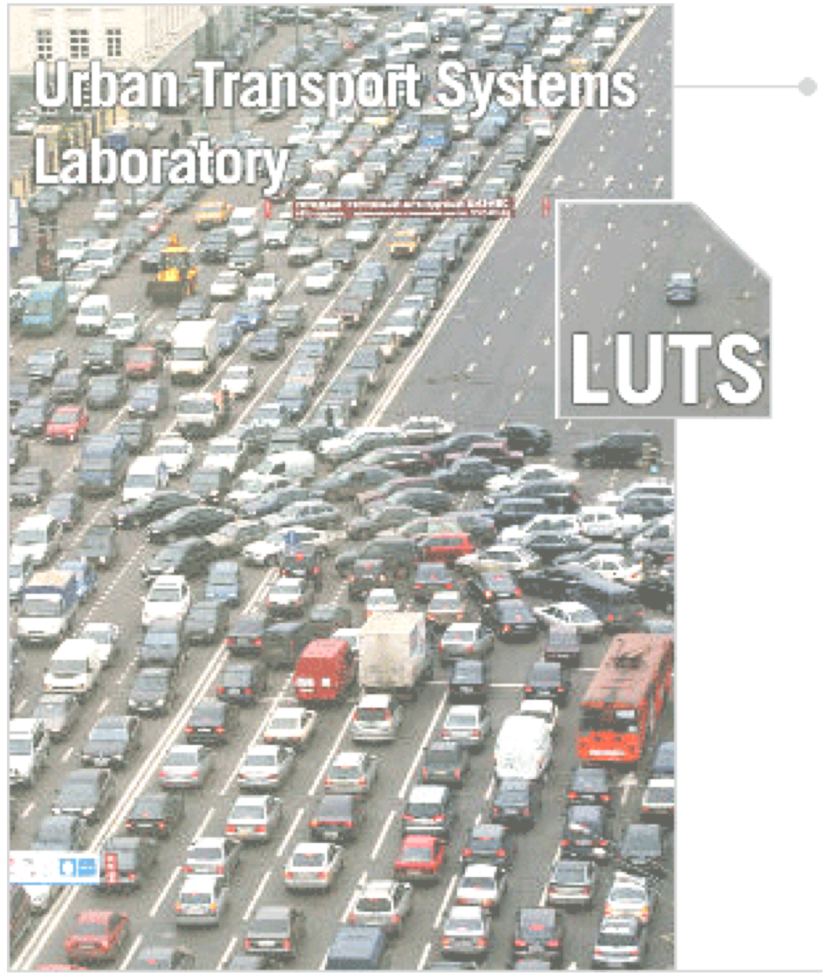
Model predictive control with multi-region MFDs (part I)

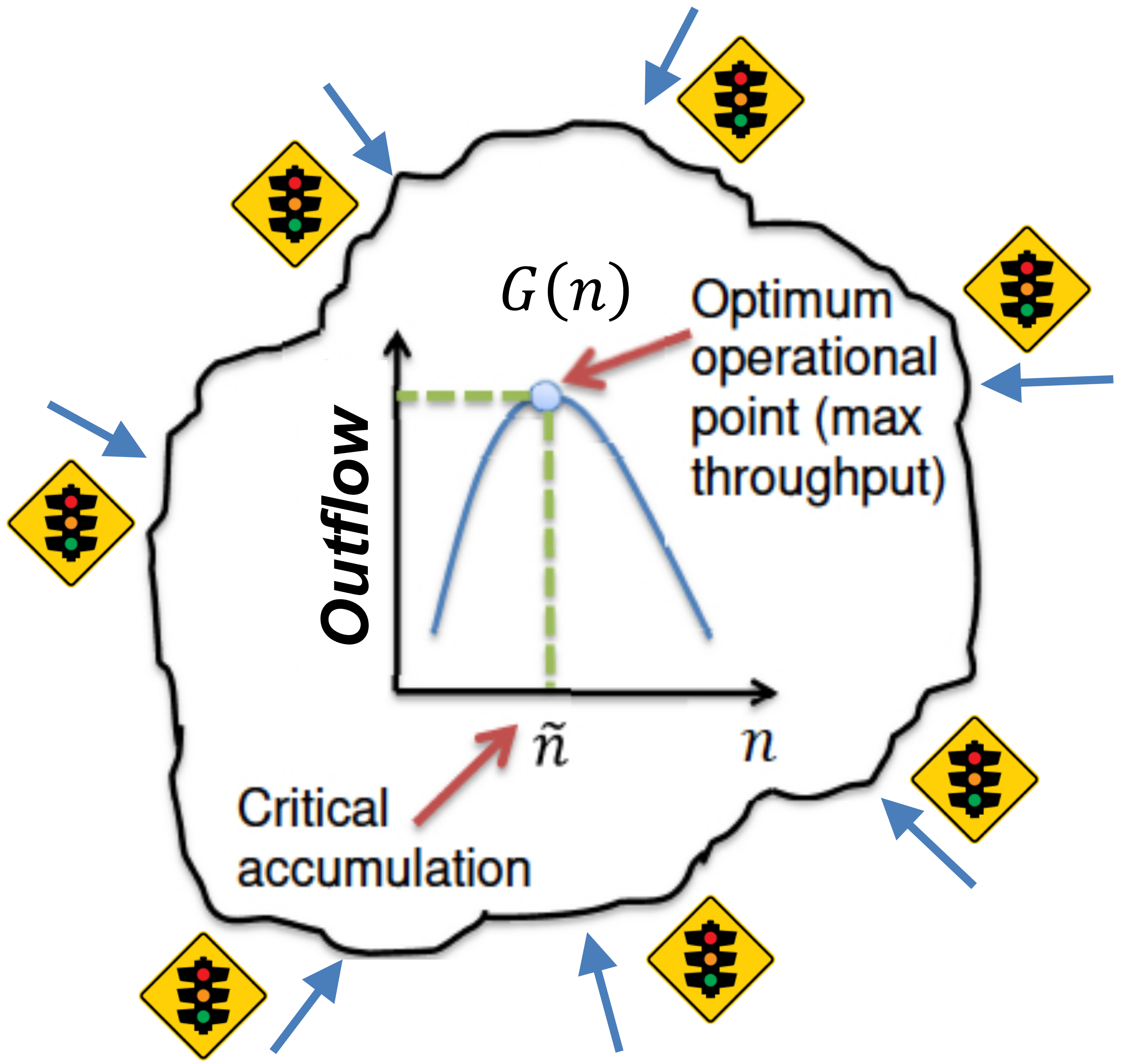
Intro to traffic flow modeling and ITS

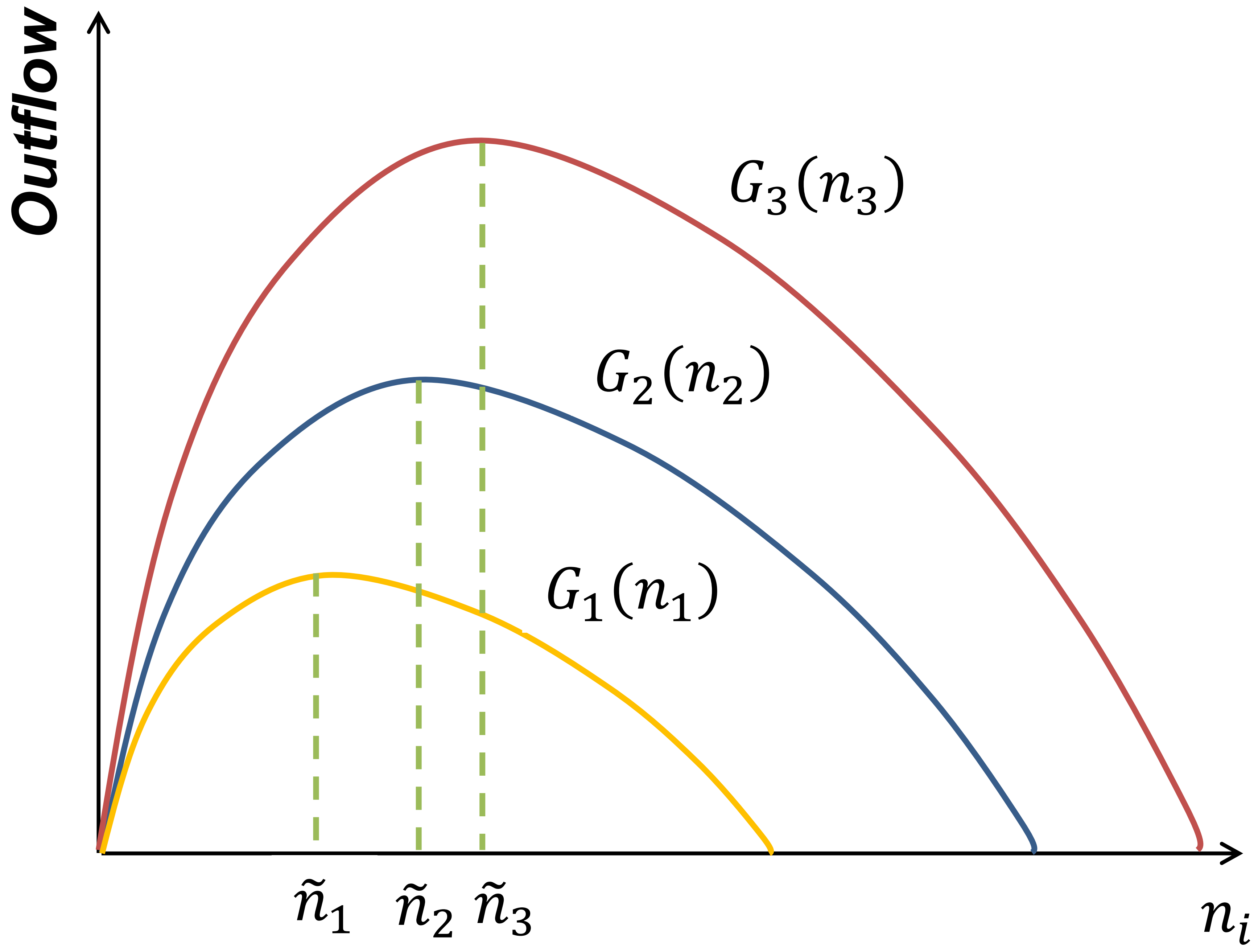
Prof. Nikolas Geroliminis

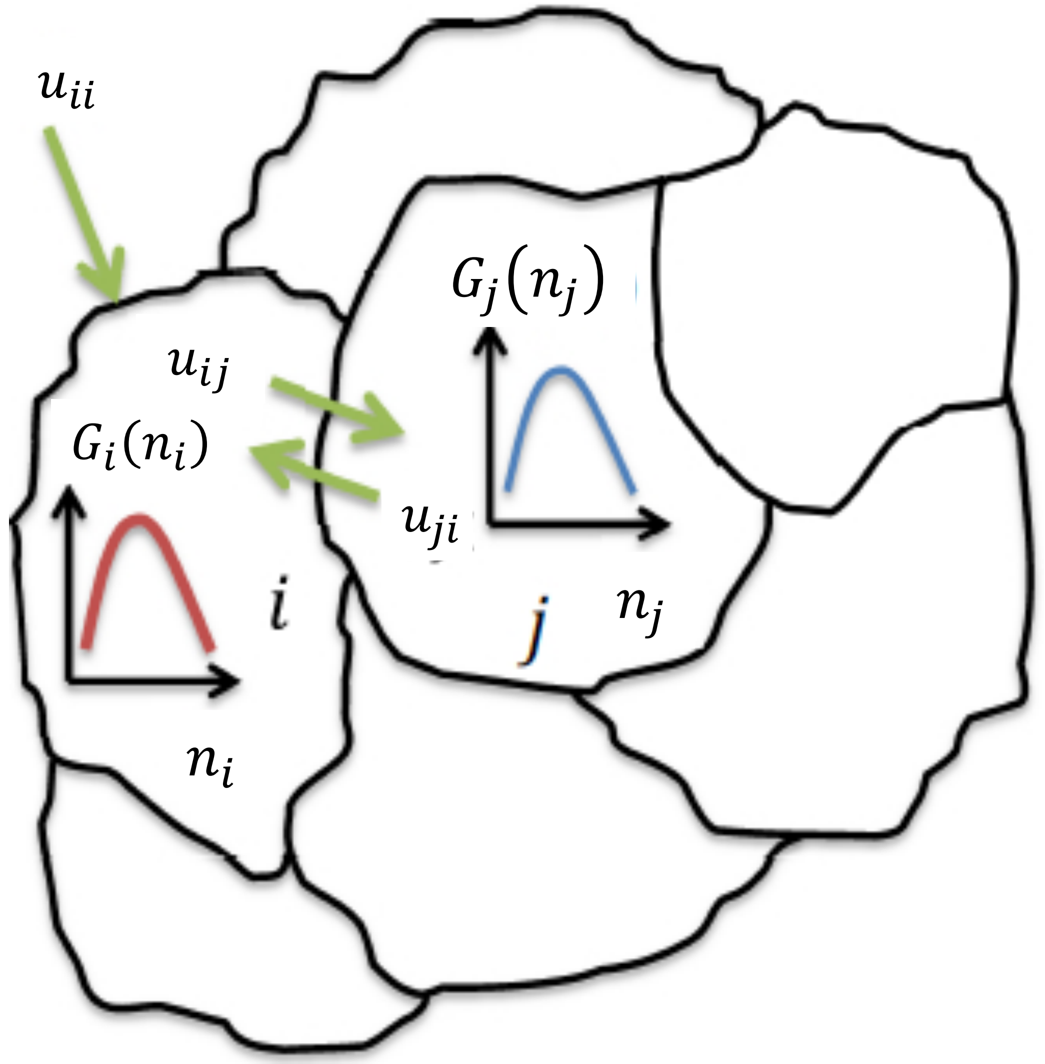


Welcome

Control logic: from single- to multi-region







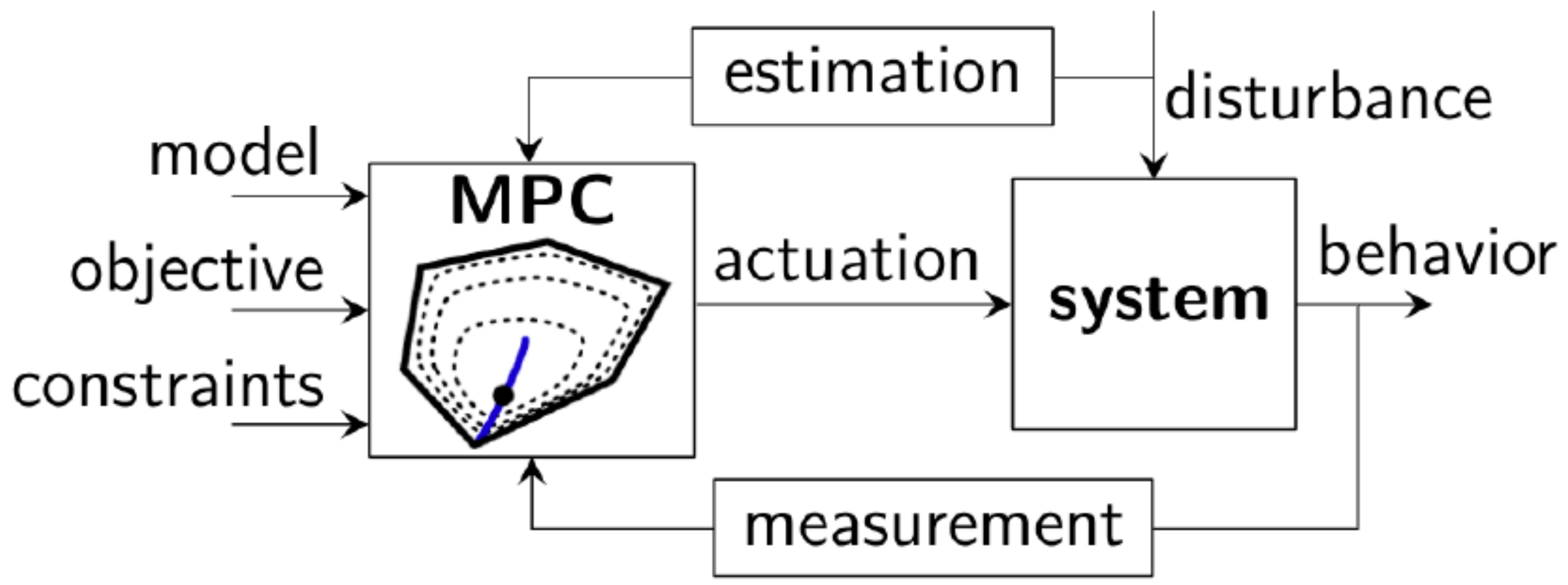
WHY MULTI-region?

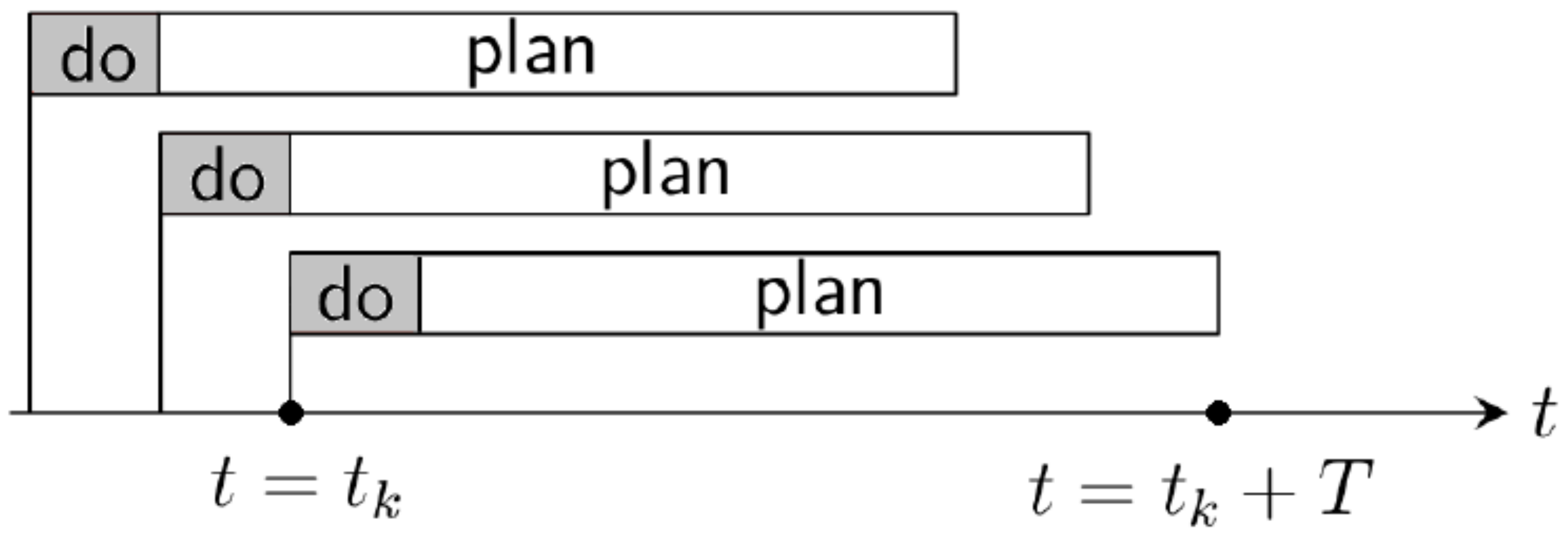
- heterogeneously congested cities with multiple pockets of congestion (partitioning)
- High internal inflows (controllability)
- Avoid long queues at boundaries (equity)

LUTS publications, 2012-2018

Why model predictive control (MPC)?

- A receding horizon procedure
- Can cope with non-linear control problems
- Can handle both state and control constraints
- Proper for real-time implementation
- Robust to prediction error and measurement noise
- Proper for traffic networks with multiple pockets of congestion

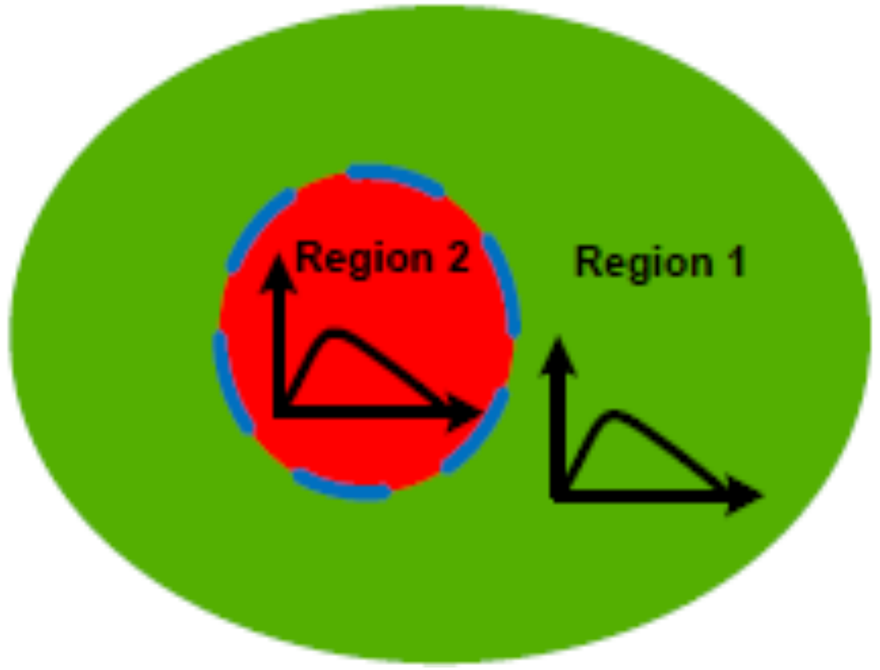




MPC for MFD systems

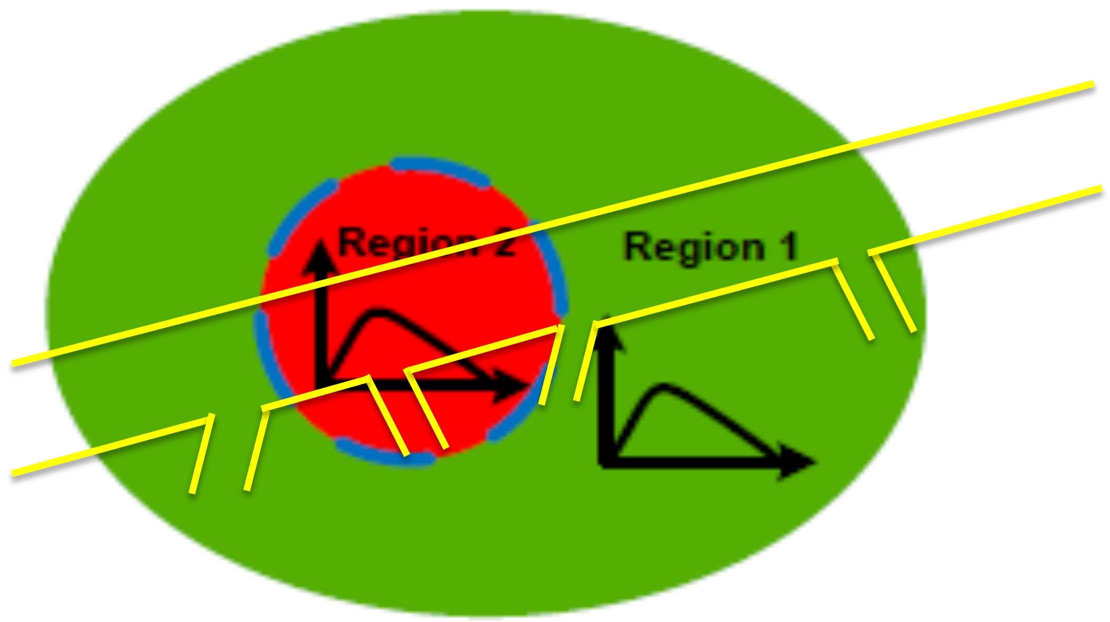
- Problem I: A two-region MFD system

Problem I



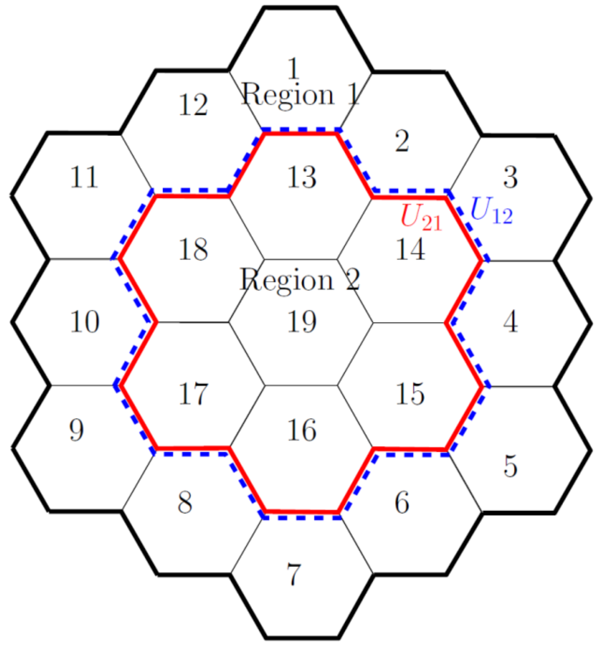
- Problem II: A mixed freeway-urban system

Problem II

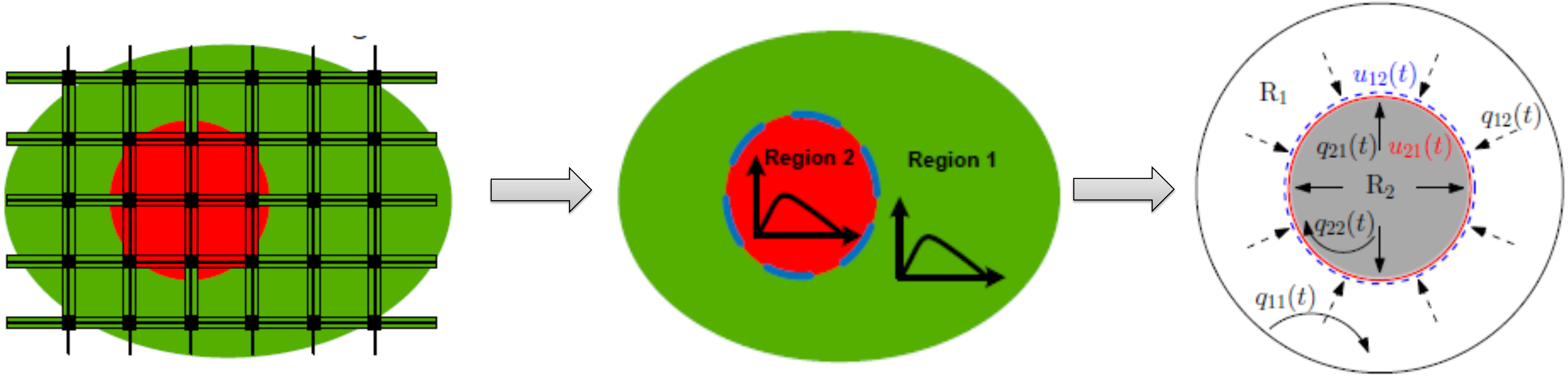


- Problem III: A multi-region MFD system with heterogeneity and control hierarchy

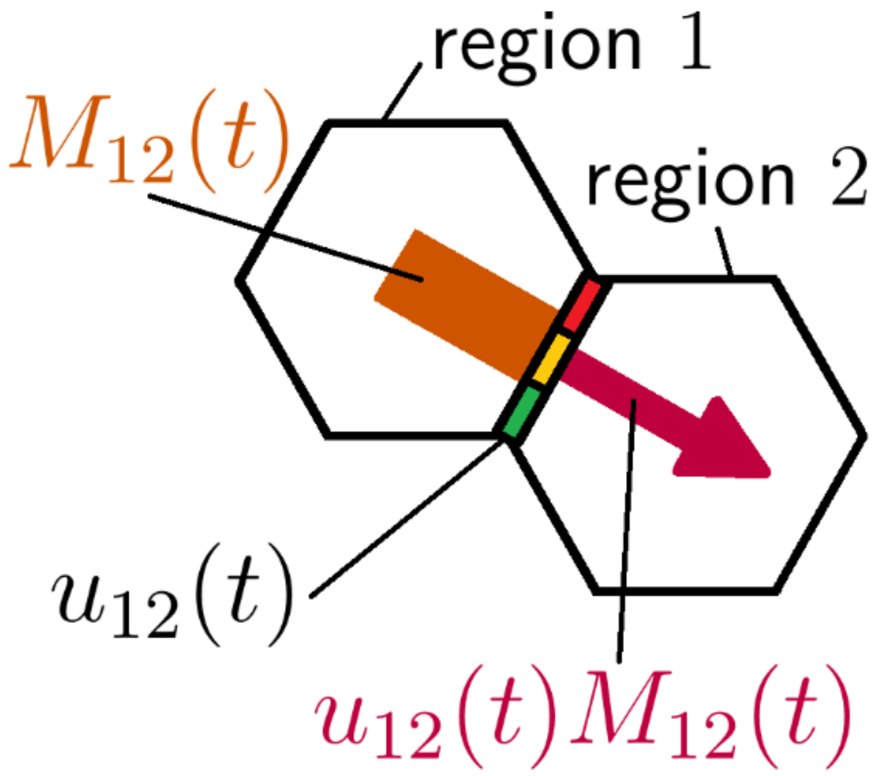
Problem III



Problem I statement



- Urban network is partitioned in homogeneous regions
- Two controllers operate on the border
- Objective function is to maximize trip endings



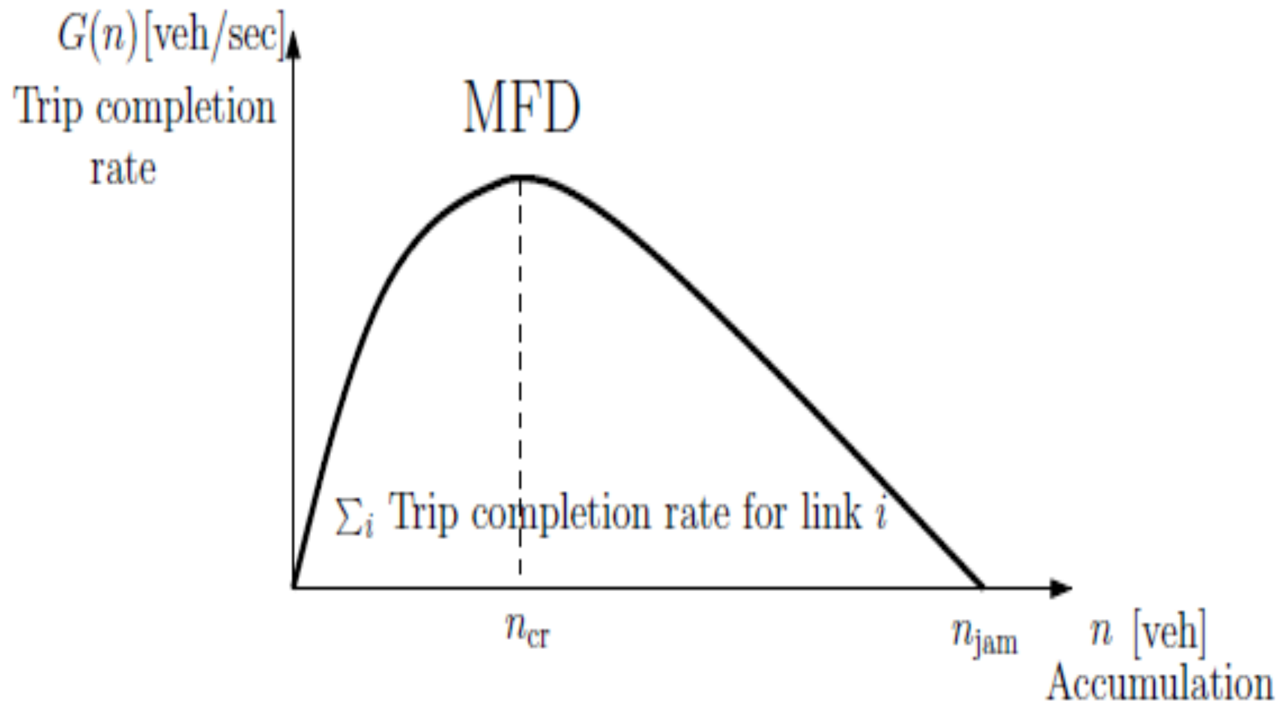
Geroliminis, Haddad, Ramezani (2013) – IEEE Trans. ITS

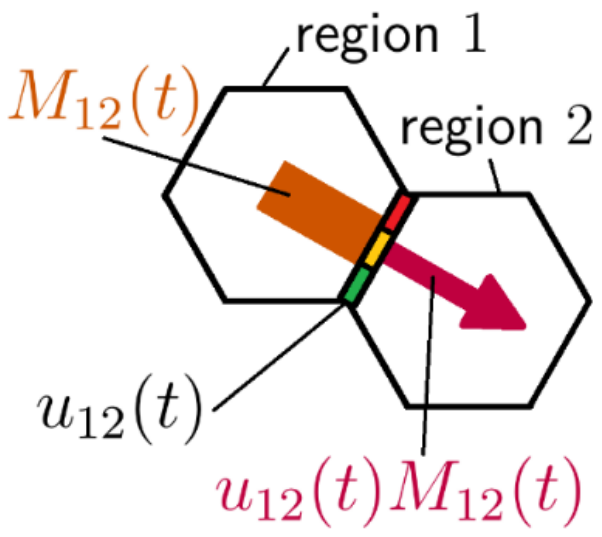
Problem I formulation

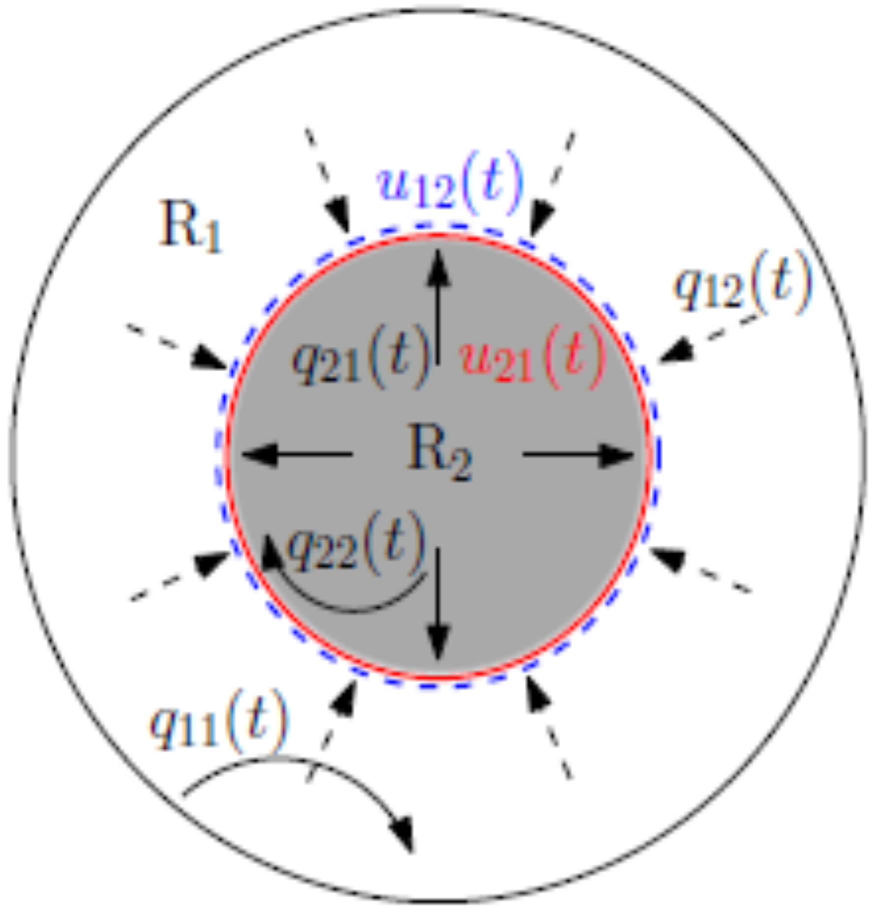
$$J = \max_{u_{12}(t), u_{21}(t)} \int_{t_0}^{t_f} [M_{11}(t) + M_{22}(t)] \mathrm{d}t$$

<u>*Subject to:*</u>

- $\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = q_{11}(t) + u_{21}(t).M_{21}(t) - M_{11}(t)$
- $\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) - u_{12}(t).M_{12}(t)$
- $\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) - u_{21}(t).M_{21}(t)$
- $\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = q_{22}(t) + u_{12}(t).M_{12}(t) - M_{22}(t)$
- $M_{ij} = \frac{n_{ij}}{n_i} \times G_i(n_i(t))$
- $u_{\min} \leq u_{12}(t) \leq u_{\max}$; $u_{\min} \leq u_{21}(t) \leq u_{\max}$
- $0 \leq n_1(t) \leq n_{1,\mathrm{jam}}$; $n_1(t) = n_{11}(t) + n_{12}(t)$
- $0 \leq n_2(t) \leq n_{2,\mathrm{jam}}$; $n_2(t) = n_{21}(t) + n_{22}(t)$





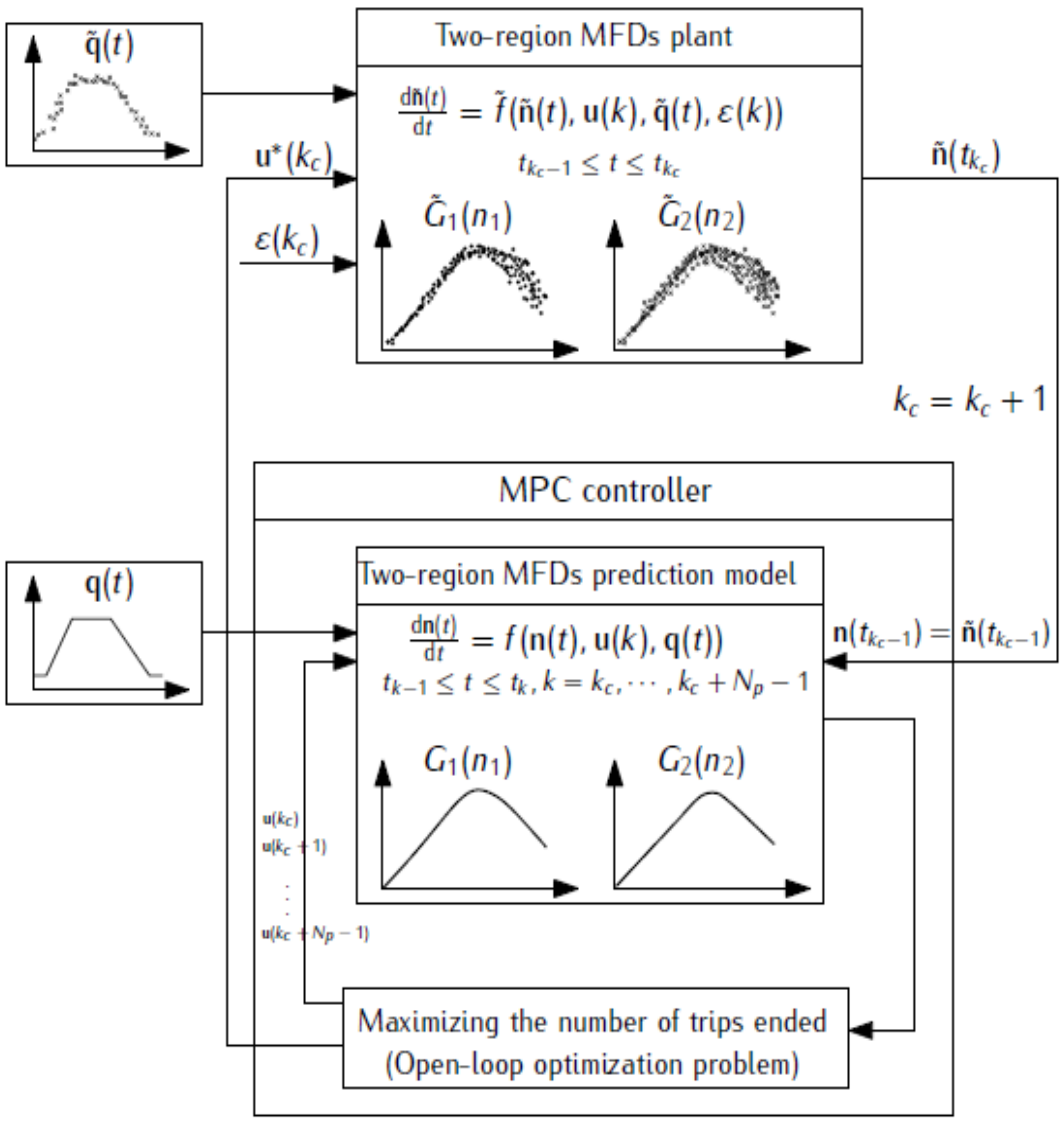


Problem I formulation

$$J = \max_{u_{12}(t), u_{21}(t)} \int_{t_0}^{t_f} [M_{11}(t) + M_{22}(t)]\mathrm{d}t$$

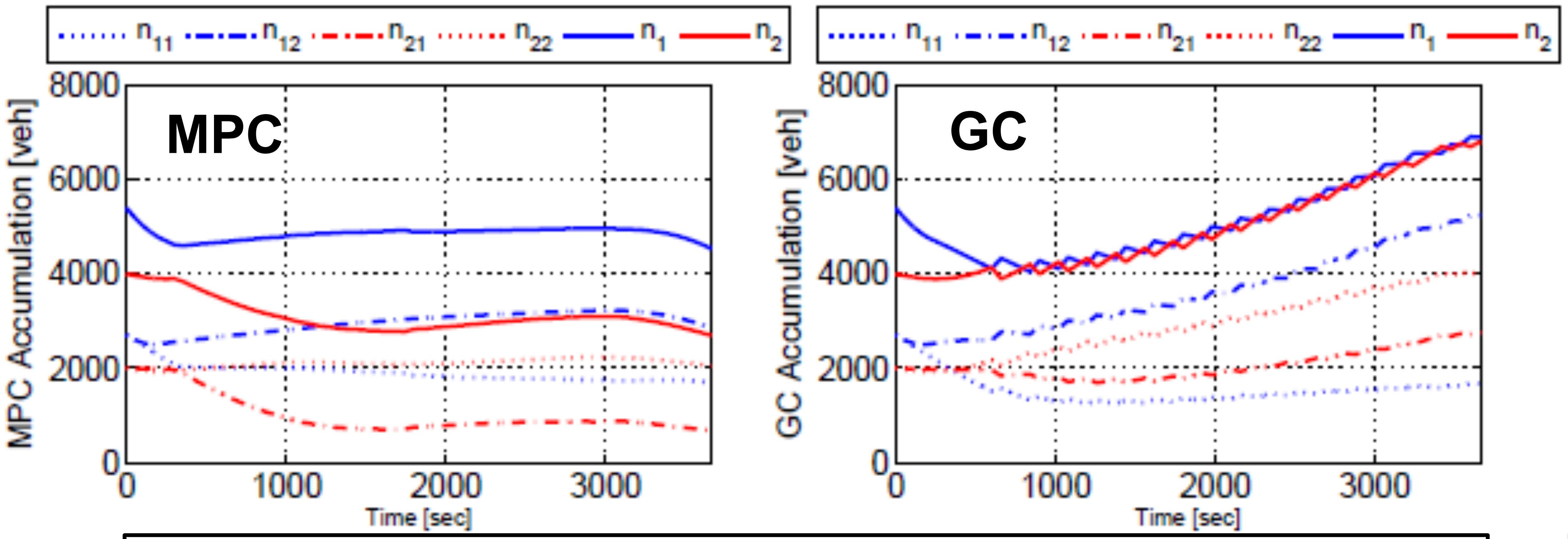
Subject to:

- $\frac{\mathrm{d}n_{11}(t)}{\mathrm{d}t} = q_{11}(t) + u_{21}(t).M_{21}(t) - M_{11}(t)$
- $\frac{\mathrm{d}n_{12}(t)}{\mathrm{d}t} = q_{12}(t) - u_{12}(t).M_{12}(t)$
- $\frac{\mathrm{d}n_{21}(t)}{\mathrm{d}t} = q_{21}(t) - u_{21}(t).M_{21}(t)$
- $\frac{\mathrm{d}n_{22}(t)}{\mathrm{d}t} = q_{22}(t) + u_{12}(t).M_{12}(t) - M_{22}(t)$
- $M_{ij} = \frac{n_{ij}}{n_i} \times G_i(n_i(t))$
- $u_{\min} \leq u_{12}(t) \leq u_{\max}$; $u_{\min} \leq u_{21}(t) \leq u_{\max}$
- $0 \leq n_1(t) \leq n_{1,\mathrm{jam}}$; $n_1(t) = n_{11}(t) + n_{12}(t)$
- $0 \leq n_2(t) \leq n_{2,\mathrm{jam}}$; $n_2(t) = n_{21}(t) + n_{22}(t)$

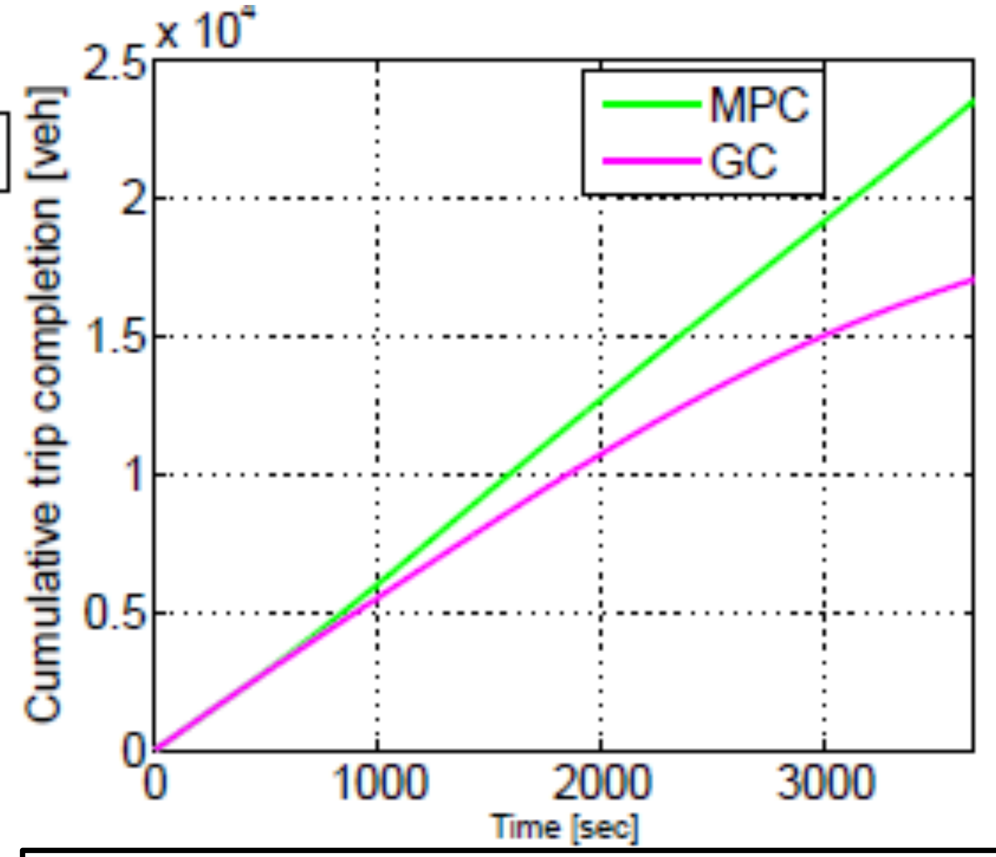


Problem I Results (no errors in MFDs)

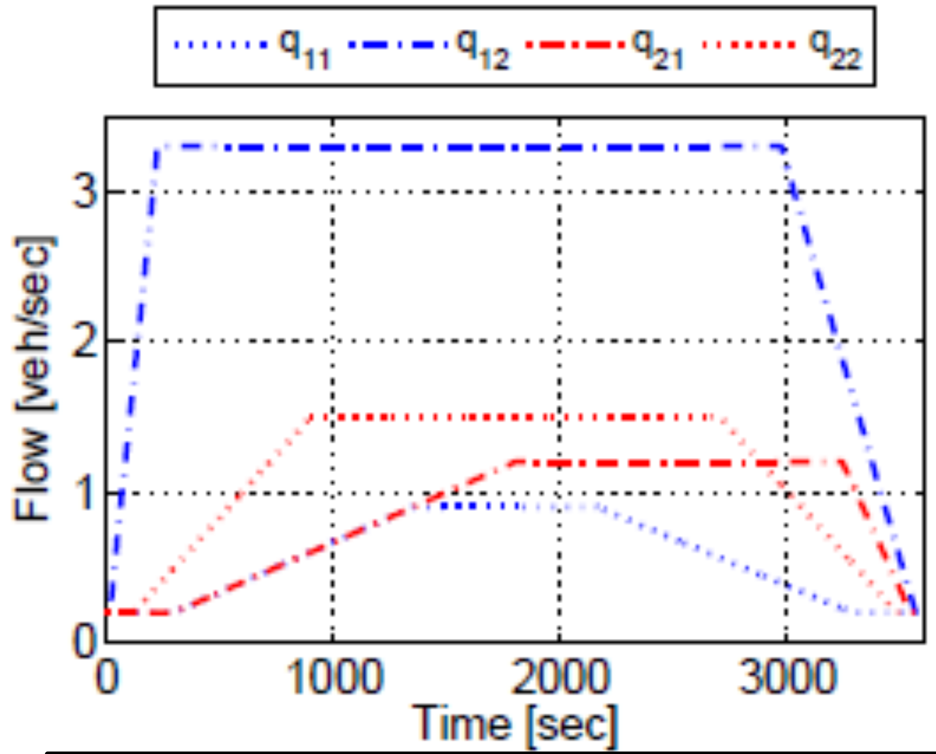
EPFL



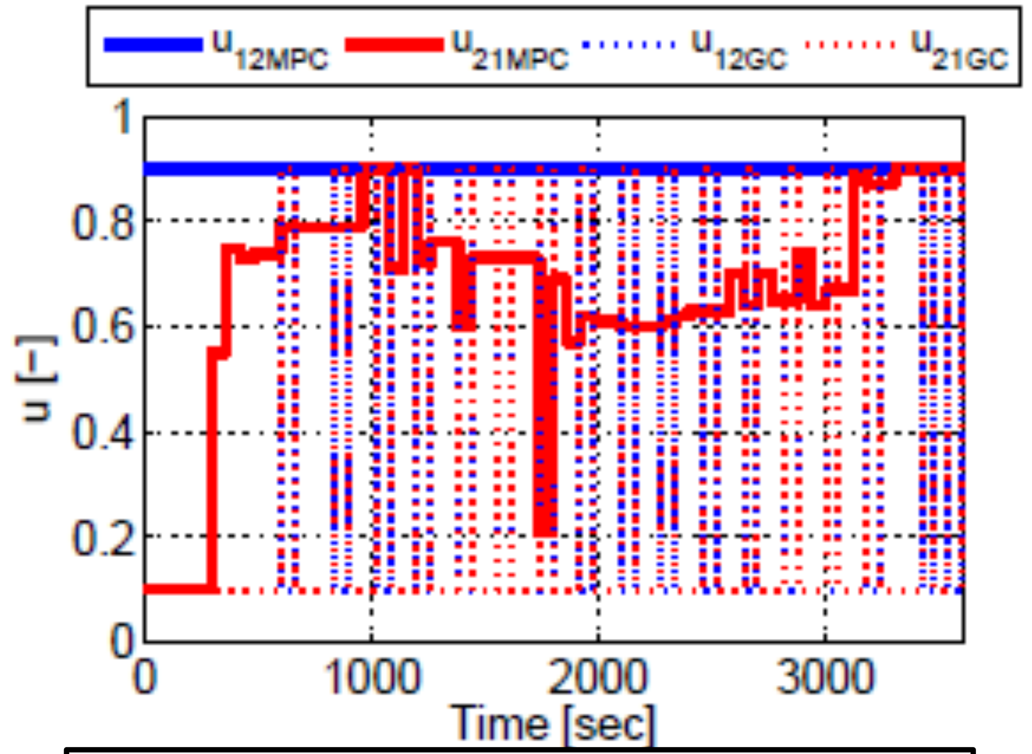
Accumulations with time



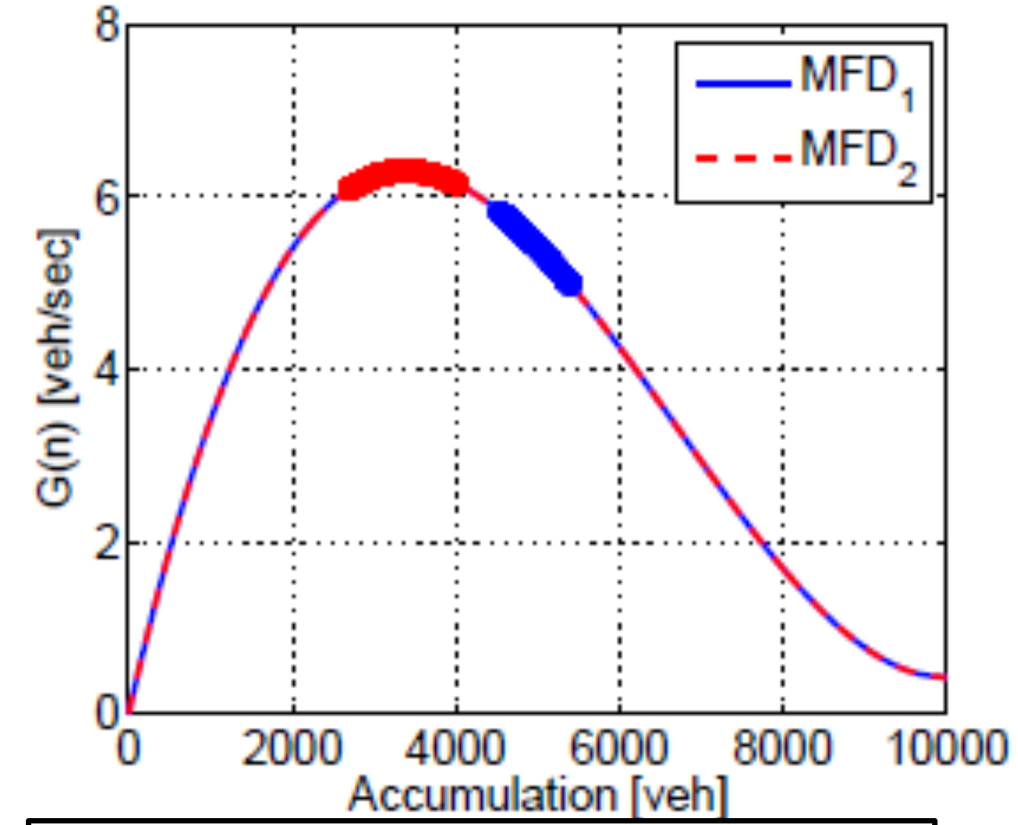
Cumulative trip endings



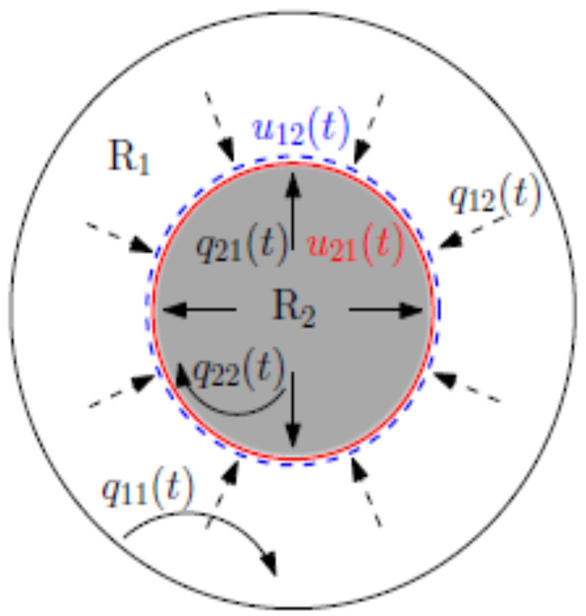
Demand



Control actions

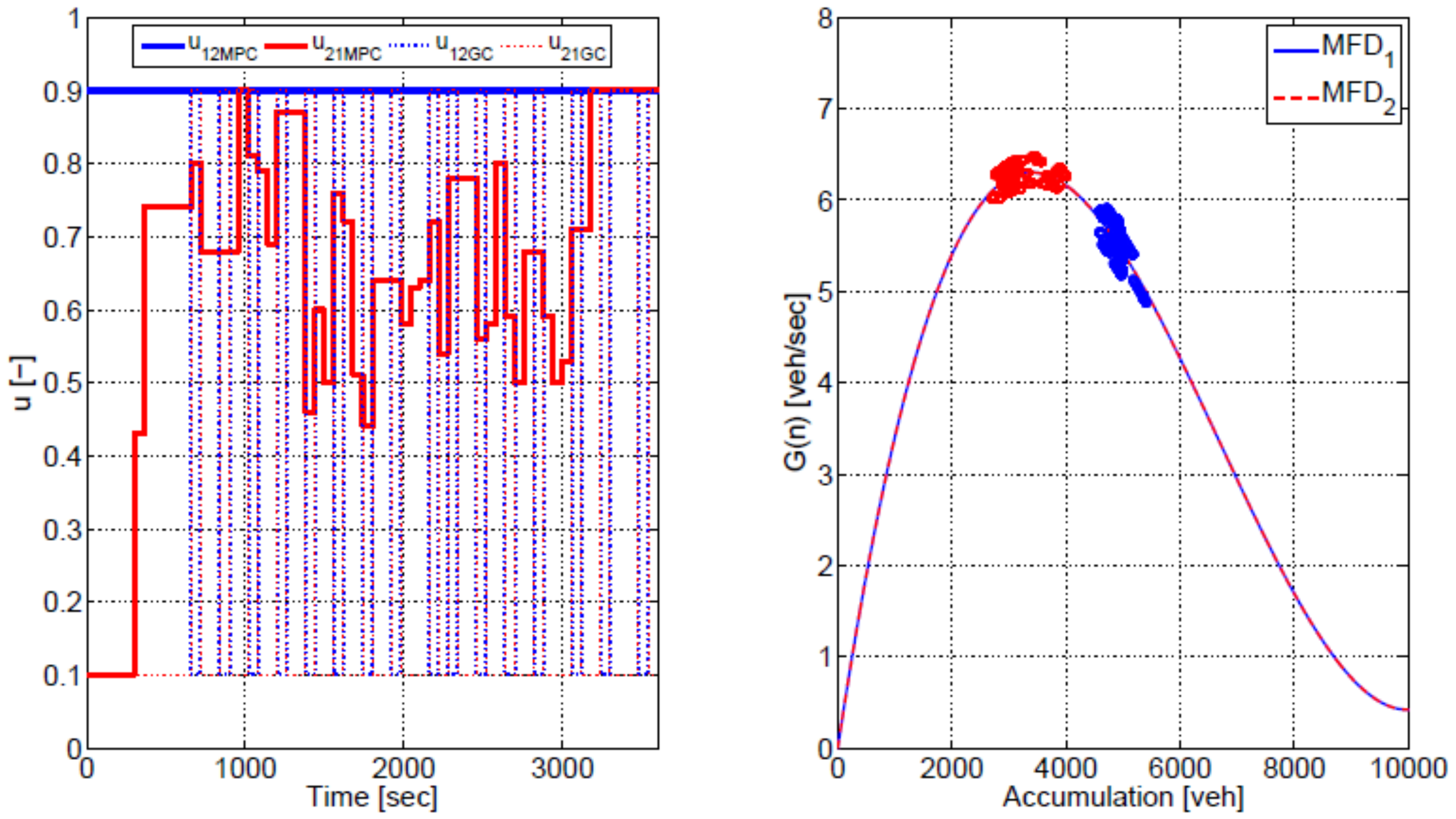


MFD states



GC: Greedy Controller

Problem I Results (with errors in MFDs)



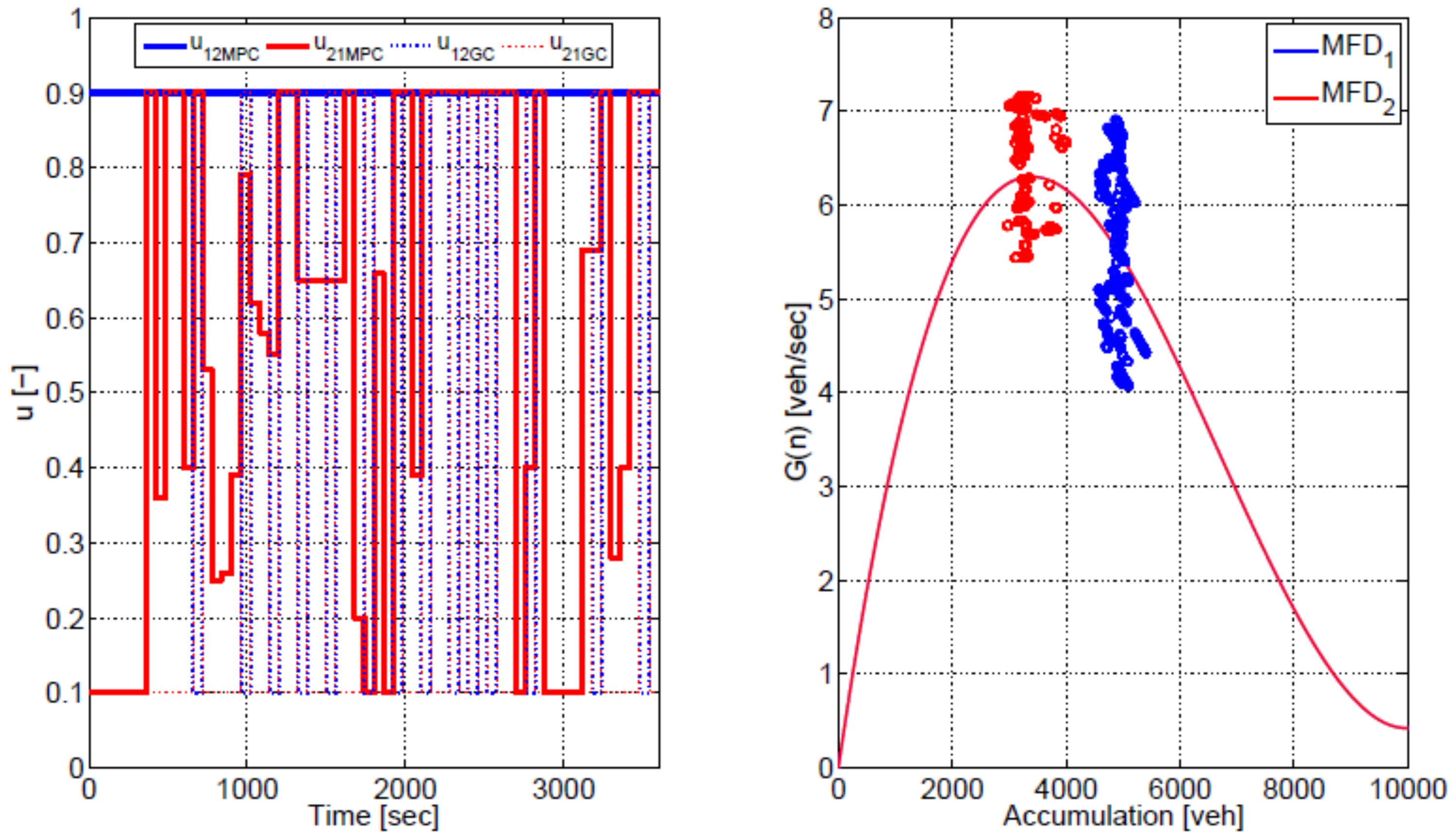
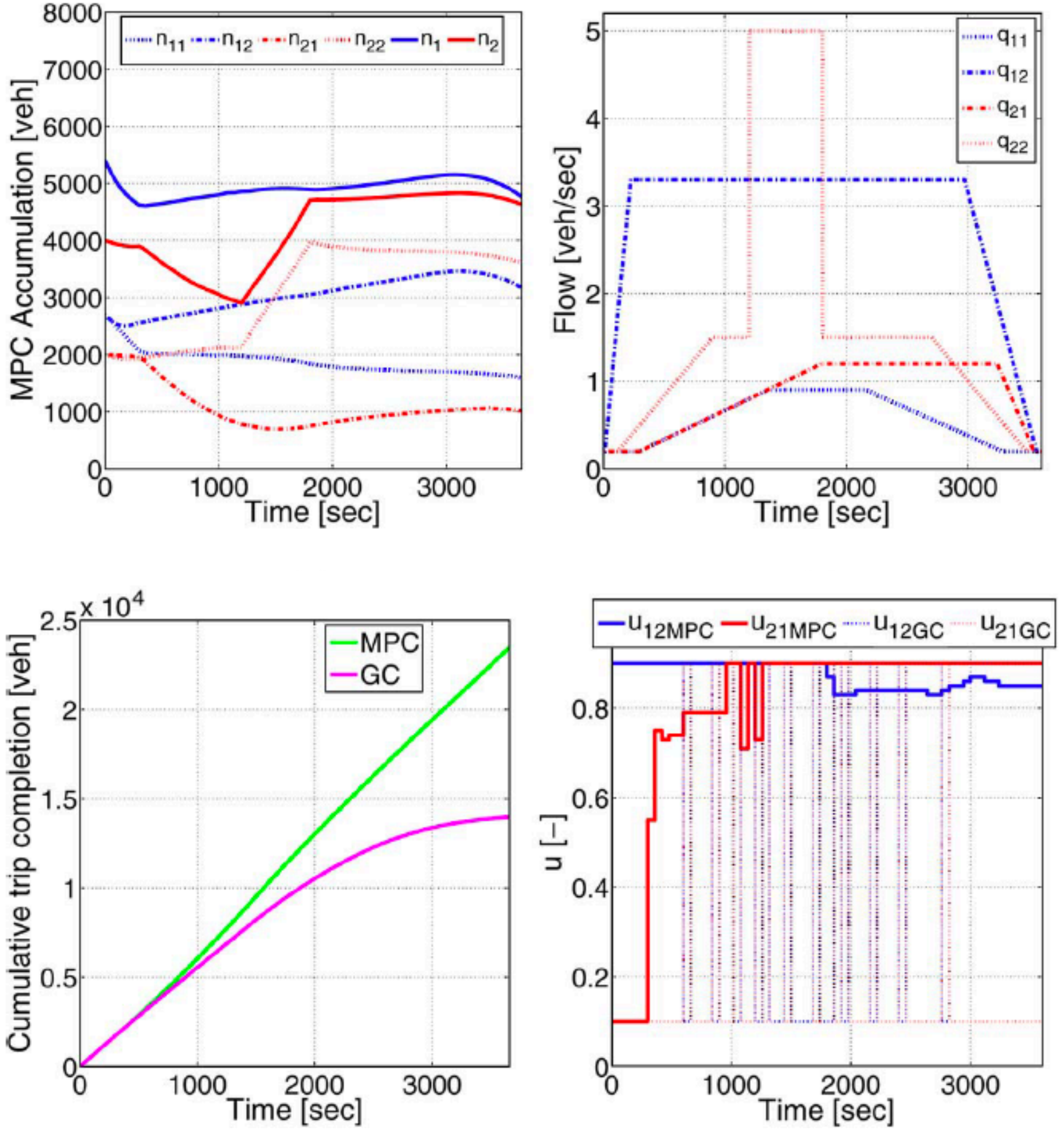


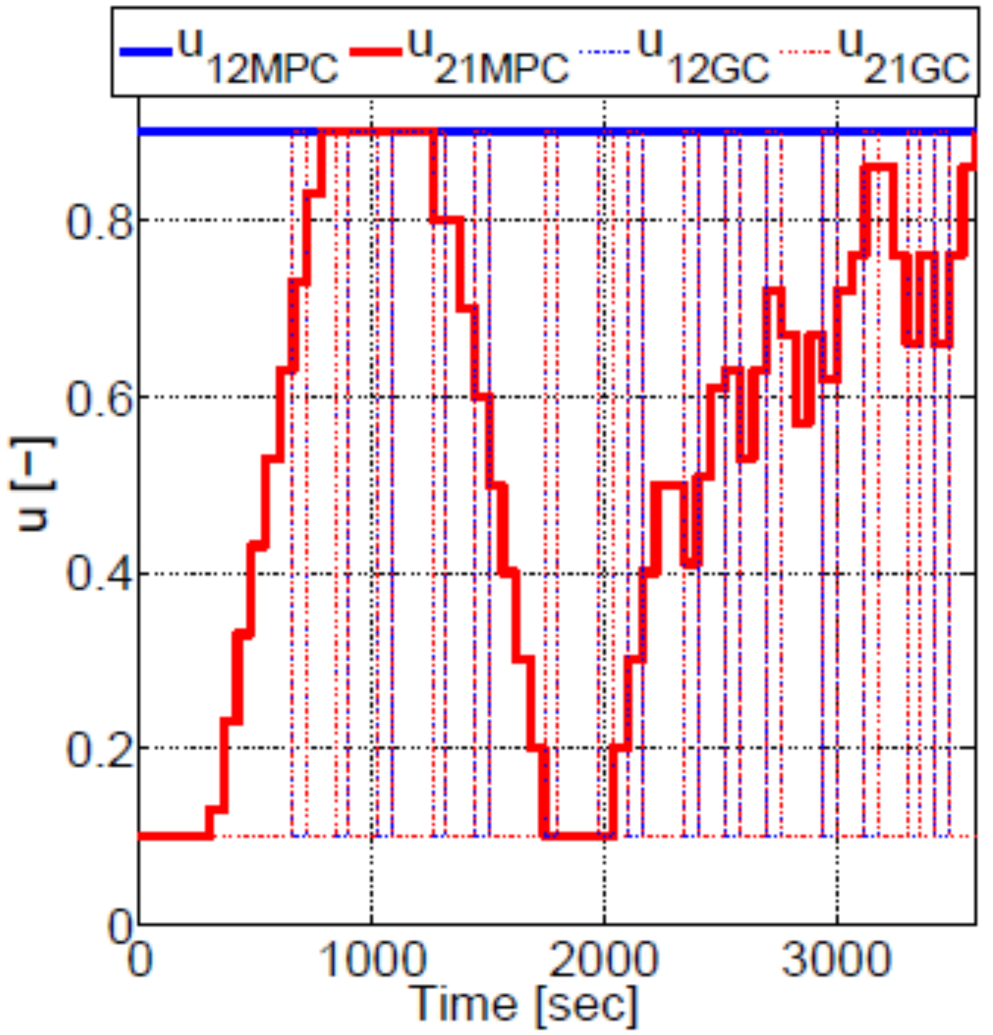
Table 1: The difference between the trip completion corresponding to MPC and greedy control $\cdot 10^3$[veh] without errors, with small and large errors in the plant MFDs.

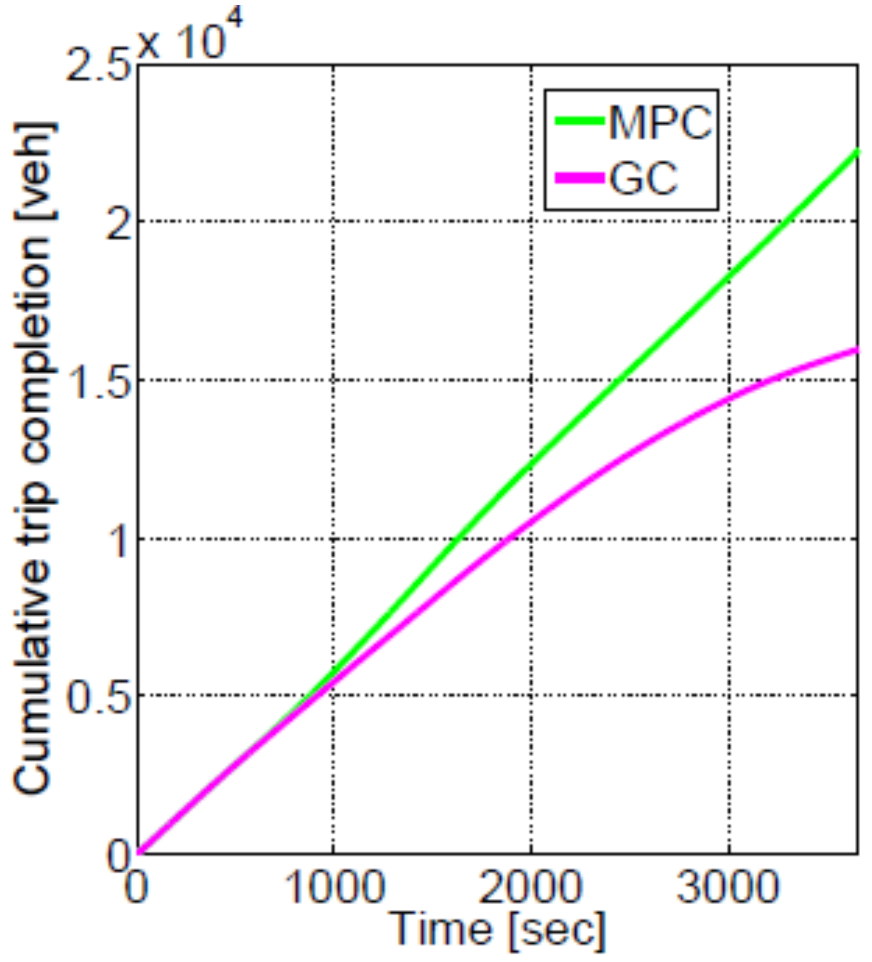
	Example	without errors ($\alpha_1 = \alpha_2 = 0$)	small errors ($\alpha_1 = \alpha_2 = 0.2$)	large errors ($\alpha_1 = \alpha_2 = 1$)
High demand	1	7791.6 (%22.5)	7847.8 (%22.7)	8112.8 (%23.5)
Medium demand	2	6536.8 (%17.3)	6530.7 (%17.3)	6556.1 (%17.4)
Low demand	3	1789.3 (%4.4)	1733.1 (%4.3)	1670.7 (%4.1)

Biased Demand / Control smoothing



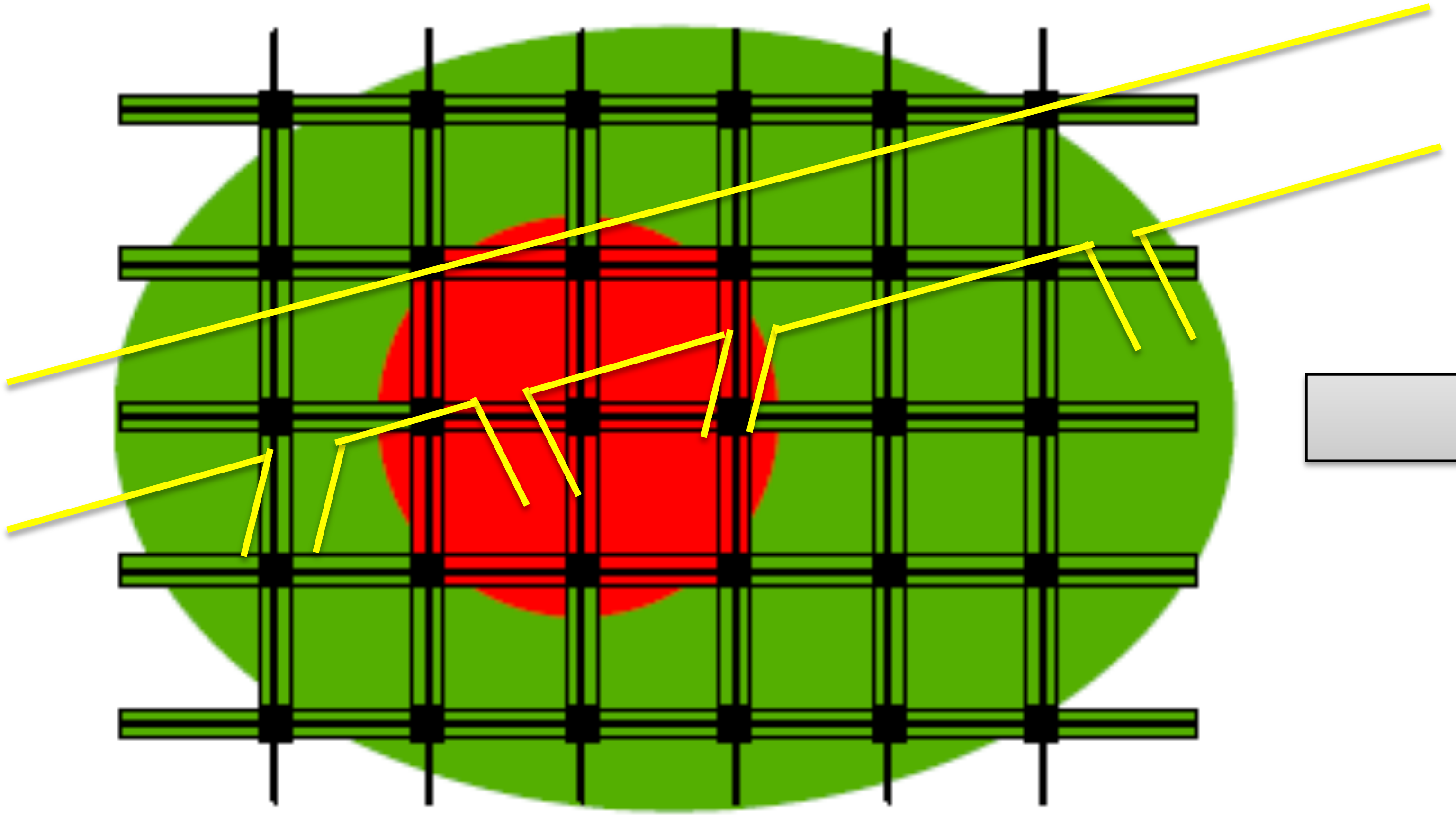
Control is not sensitive to unknown demand bias

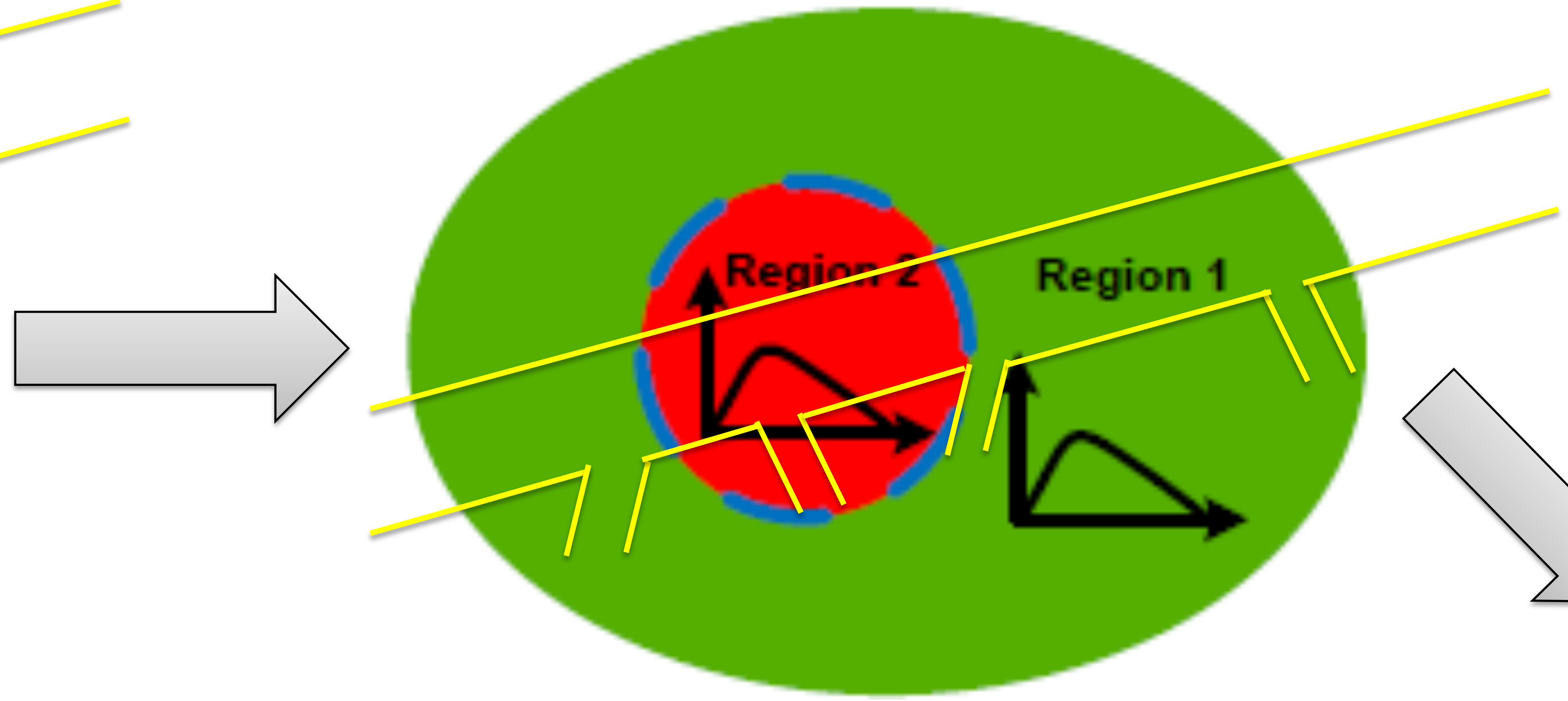


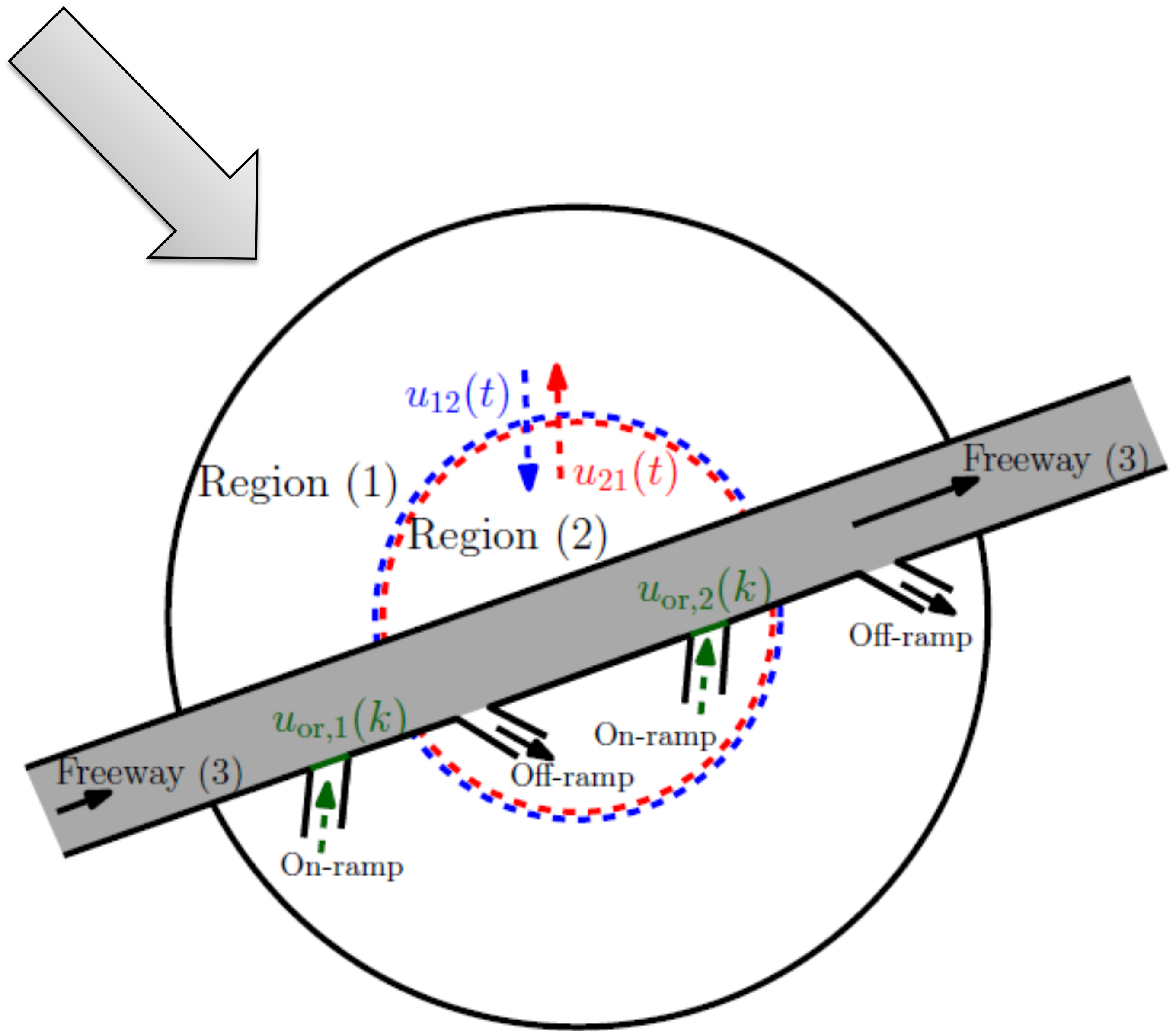


Smoothing control by imposing constraints in the jump between *t* and *t+1*

Problem II statement







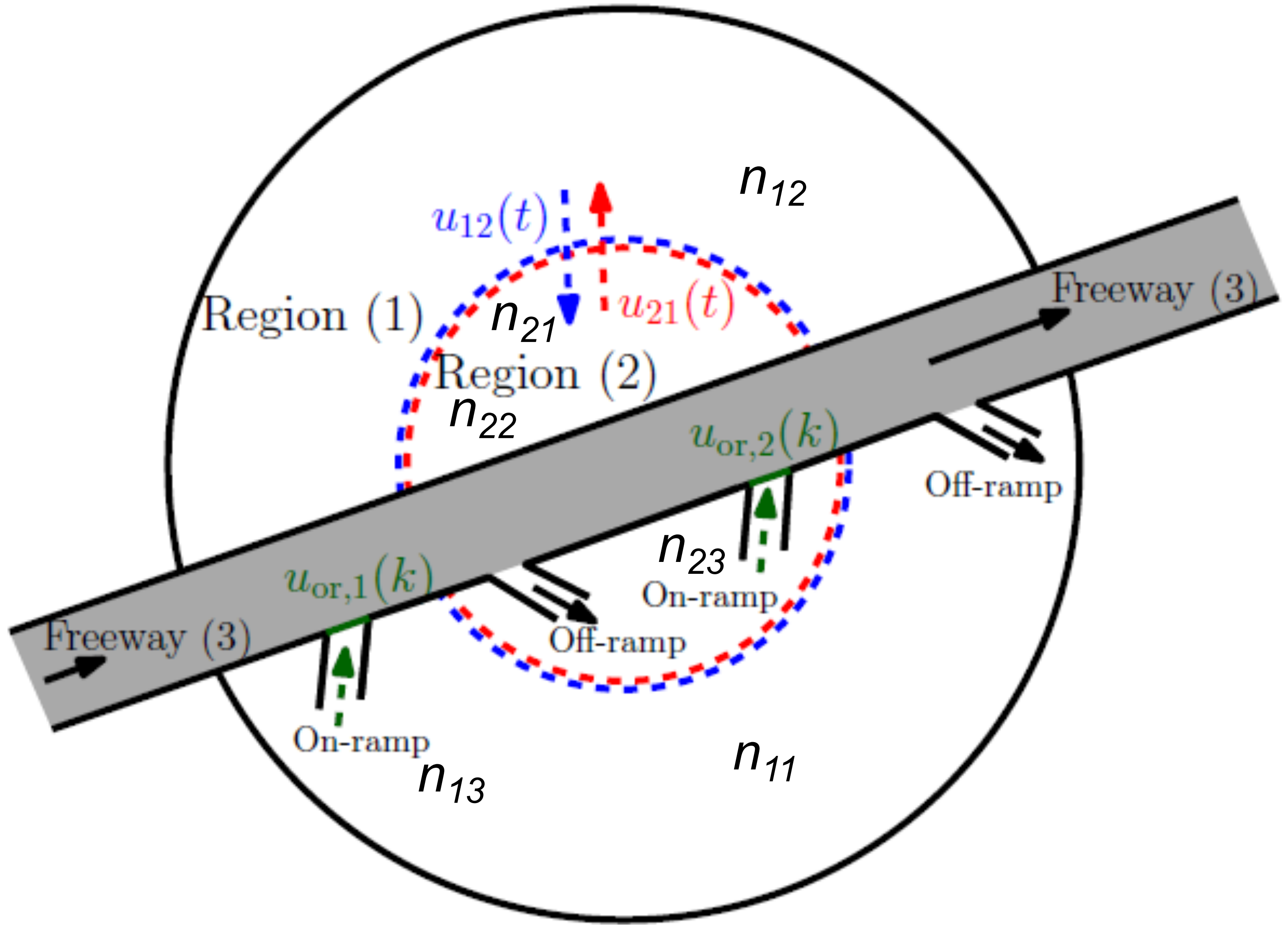
- **<u>Coordinated</u>** control of a mixed freeway-arterial network
- Objective is to minimize the network total delay

❑ **Controllers are:**
  - <u>2 Ramp metering</u>
  - <u>2 Perimeter controllers</u>

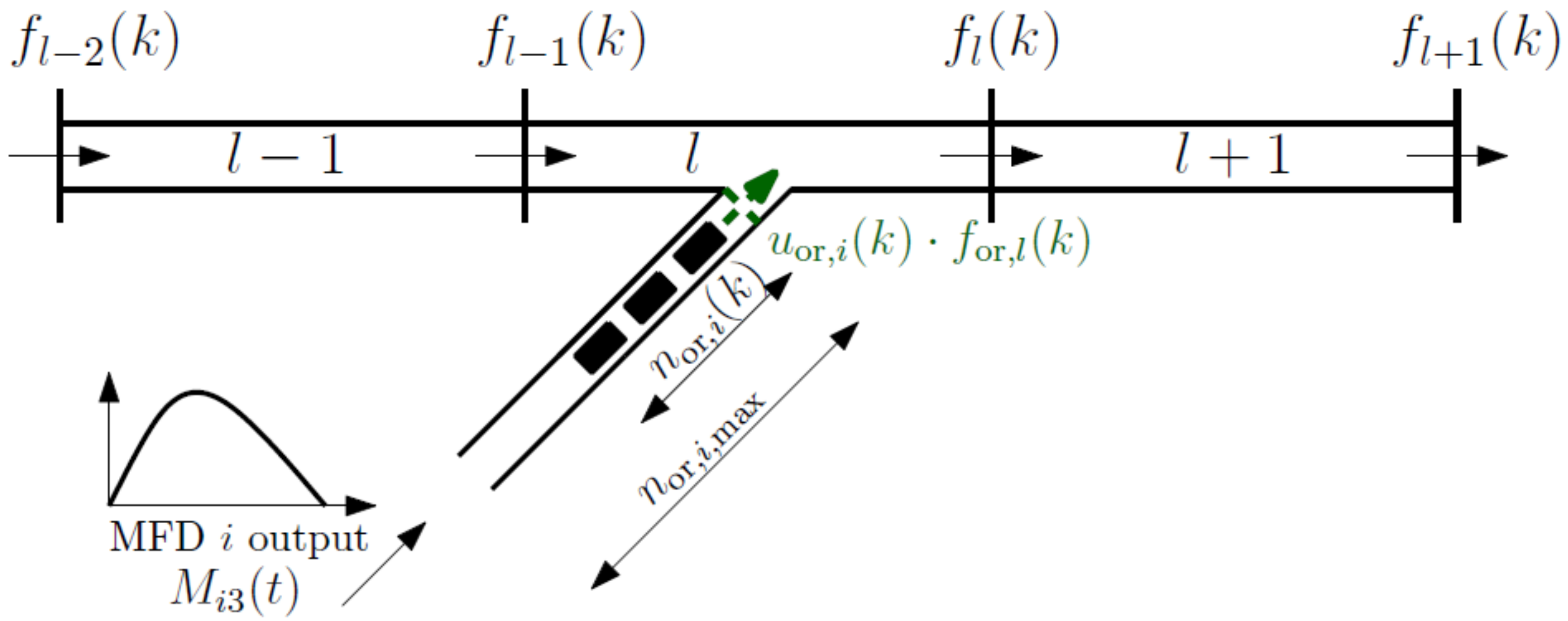
Haddad, Ramezani, Geroliminis (2013) – Trans. Res. part B

Problem II system characteristics

- The urban regions are modeled with MFDs
- The traffic dynamics of the freeway are modeled with the asymmetric cell transmission model (ACTM)
- Dynamic trip route choice
- Time dependent O-Ds



O\ D	1	2	3
1	$q_{11}: 1 \to 1$ $q_{131}: 1 \to 3 \to 1$	$q_{12}: 1 \to 2$ $q_{132}: 1 \to 3 \to 2$	$q_{13}: 1 \to 3$ $q_{123}: 1 \to 2 \to 3$
2	$q_{21}: 2 \to 1$ $q_{231}: 2 \to 3 \to 1$	$q_{22}: 2 \to 2$	$q_{23}: 2 \to 3$ $q_{213}: 2 \to 1 \to 3$
3	$q_{31}: 3 \to 1$ $q_{321}: 3 \to 2 \to 1$	$q_{32}: 3 \to 2$ $q_{312}: 3 \to 1 \to 2$	$q_{33}: 3 \to 3$



Problem II formulation

$$J = \min_{\substack{u_{12}(k), u_{21}(k), \\ u_{on,1}(k), u_{on,2}(k); \\ \text{for } k=0,\dots,K-1}} T_k \cdot \left[\sum_{k=0}^{K-1} (n_1(k) + n_2(k))\right] + \left[\sum_{k=0}^{K-1}\sum_{l=1}^{L} x_l(k) + \sum_{k=0}^{K-1}\sum_{i=1}^{2} n_{on,i}(k)\right]$$

Urban system	Freeway system

subject to

State dynamics

$$n_{11}(k+1) = n_{11}(k) + T_k \cdot \left[\frac{\hat{q}_{321}(k) + q_{21}(k)}{\hat{q}_{321}(k) + q_{213}(k) + q_{21}(k)} \cdot u_{21}(k) \cdot M_{21}(k) + q_{11}(k) + \hat{q}_{231}(k) + \hat{q}_{31}(k) - M_{11}(k)\right]$$

$$n_{12}(k+1) = n_{12}(k) + T_k \cdot [q_{12}(k) + q_{123}(k) + \hat{q}_{312}(k) - u_{12}(k) \cdot M_{12}(k)]$$

$$n_{13}(k+1) = n_{13}(k) + T_k \cdot \left[\frac{q_{213}(k)}{\hat{q}_{321}(k) + q_{213}(k) + q_{21}(k)} \cdot u_{21}(k) \cdot M_{21}(k) + q_{13}(k) + q_{131}(k) + q_{132}(k) - \min(M_{13}(k), C_{on,1}(k))\right]$$

$$n_{21}(k+1) = n_{21}(k) + T_k \cdot [q_{21}(k) + q_{213}(k) + \hat{q}_{321}(k) - u_{21}(k) \cdot M_{21}(k)]$$

$$n_{22}(k+1) = n_{22}(k) + T_k \cdot \left[\frac{q_{12}(k) + \hat{q}_{312}(k)}{q_{12}(k) + q_{123}(k) + \hat{q}_{312}(k)} \cdot u_{12}(k) \cdot M_{12}(k) + q_{22}(k) + \hat{q}_{132}(k) + \hat{q}_{32}(k) - M_{22}(k)\right]$$

$$n_{23}(k+1) = n_{23}(k) + T_k \cdot \left[\frac{q_{123}(k)}{q_{12}(k) + q_{123}(k) + \hat{q}_{321}(k)} \cdot u_{12}(k) \cdot M_{12}(k) + q_{23}(k) + q_{231}(k) - \min(M_{23}(k), C_{on,2}(k))\right]$$

State constraints

$$0 \leqslant \sum_{j=1}^{3} n_{ij}(k) \leqslant n_{i,jam} \quad i = 1,2$$

Control constraints

$$u_{min} \leqslant u_{ij}(k) \leqslant u_{max} \quad i = 1,2; \; j = 3 - i$$

$$u_{min} \leqslant u_{on,i}(k) \leqslant u_{max} \quad i = 1,2$$

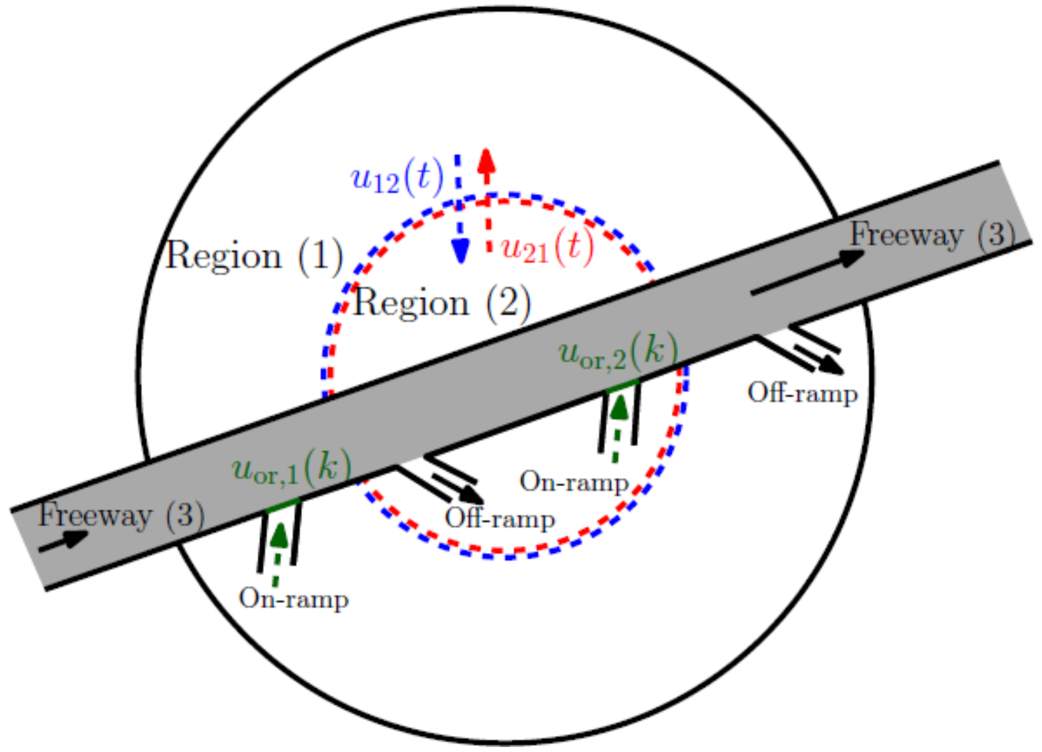
Initial conditions

$$n_{ij}(0) = n_{ij,0} \quad i = 1,2; \; j = 1,2,3$$

$$x_l(0) = x_{l,0} \quad l = 1,2,\dots,L$$

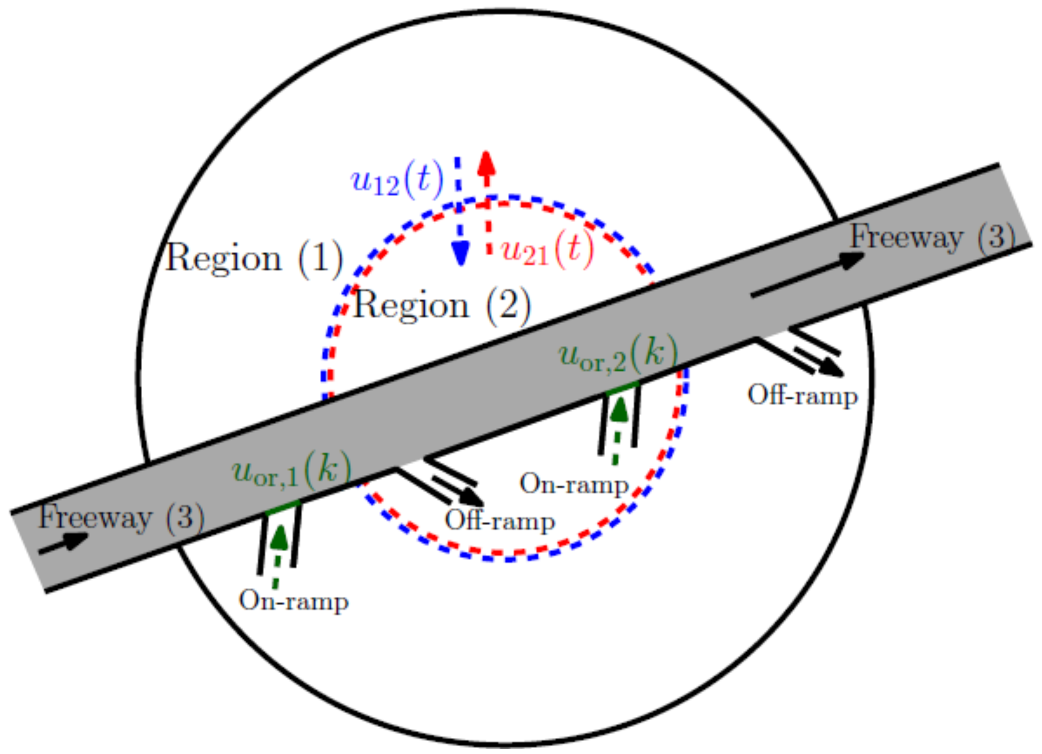
$$n_{on,i}(0) = n_{on,i,0} \quad i = 1,2$$

+ ACTM freeway traffic state dynamics



Performance of different controllers

1. **ALINEA Ramp metering:**
$$f_{or,l}(k+1) = f_{or,l}(k) + \kappa\big(x_l(k) - x_{ref}\big)$$
2. **Urban MPC + ALINEA**
3. **Decentralized MPC (Urban MPC + Freeway MPC)**
4. **Centralized MPC (Network MPC)**

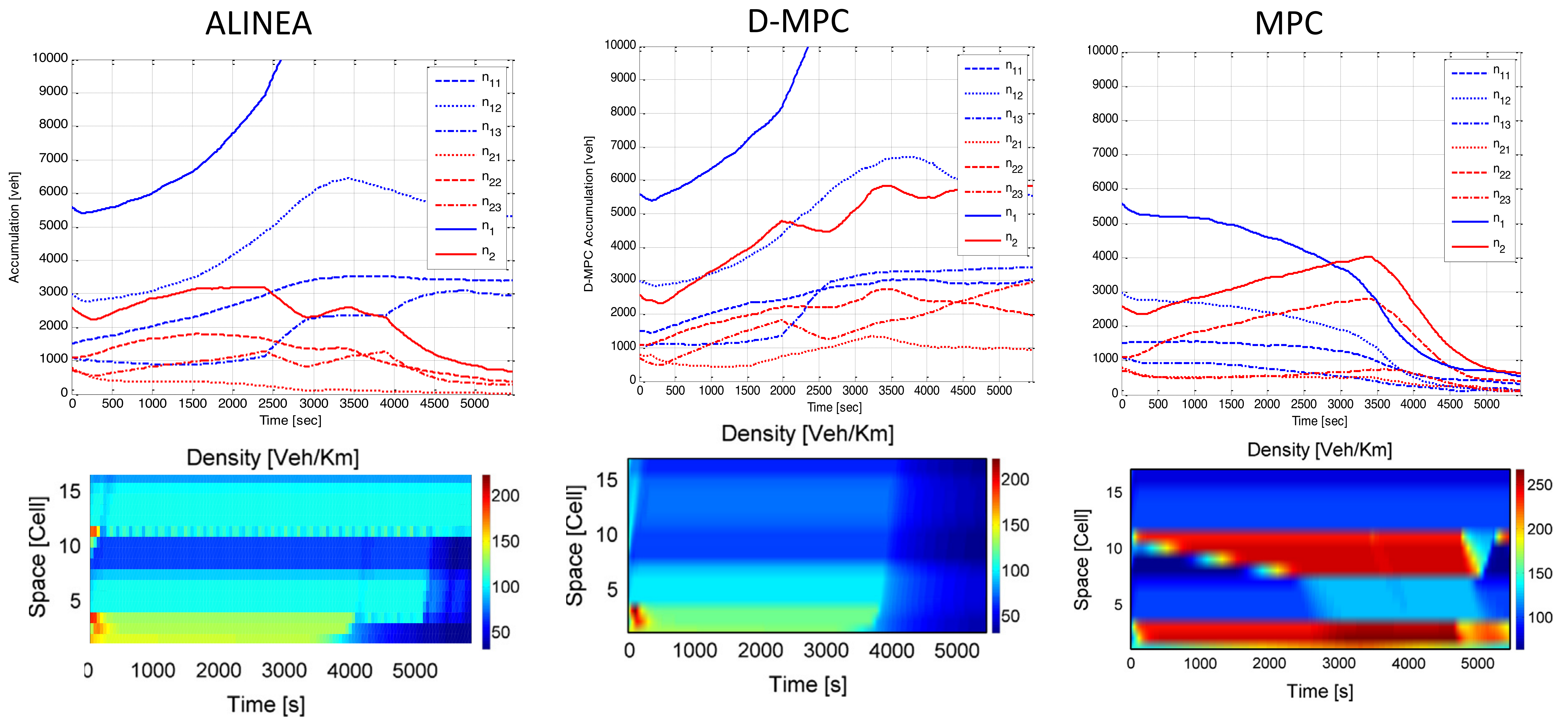


	ALINEA	Urban MPC + ALINEA	D-MPC	MPC
Region 1	**524**	**504**	**537**	**196**
Region 2	**121**	**162**	**279**	**143**
Freeway	**129**	**129**	**124**	**218**
On-Ramp 1	**13**	**13**	**16**	**8**
On-Ramp 2	**16**	**16**	**16**	**16**
Network	**803**	**823**	**972**	**581**

Performance of different controllers

Urban Accumulation [veh]

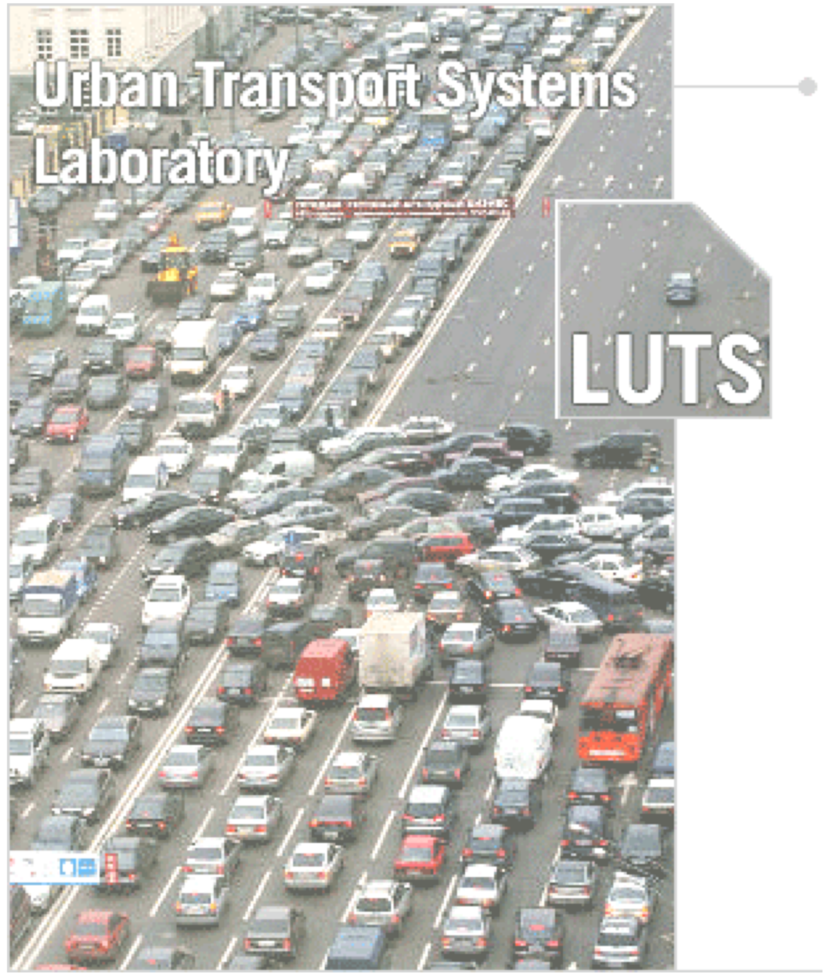
FRWY



Model predictive control with multi-region MFDs (part II)

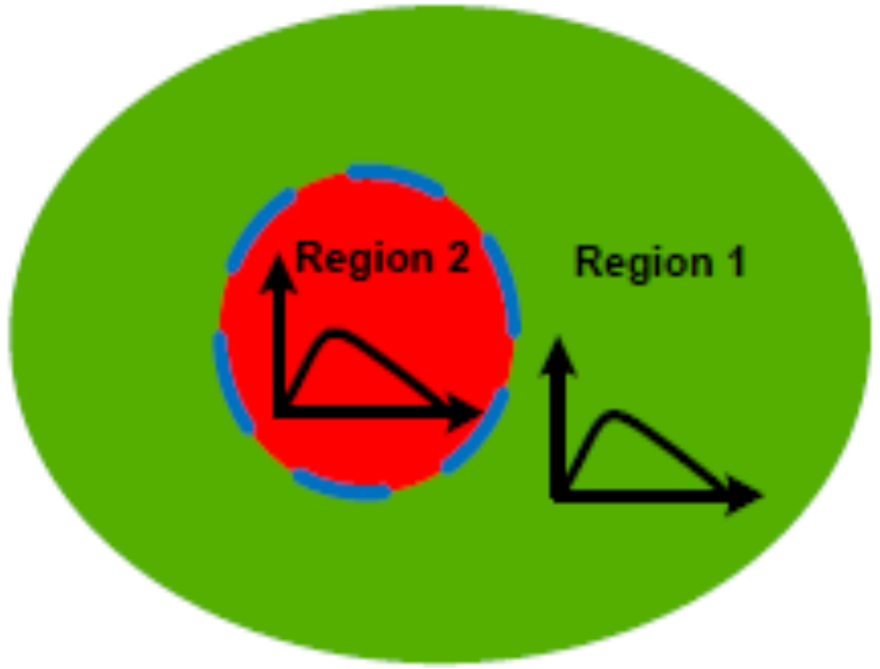
Intro to traffic flow modeling and ITS

Prof. Nikolas Geroliminis

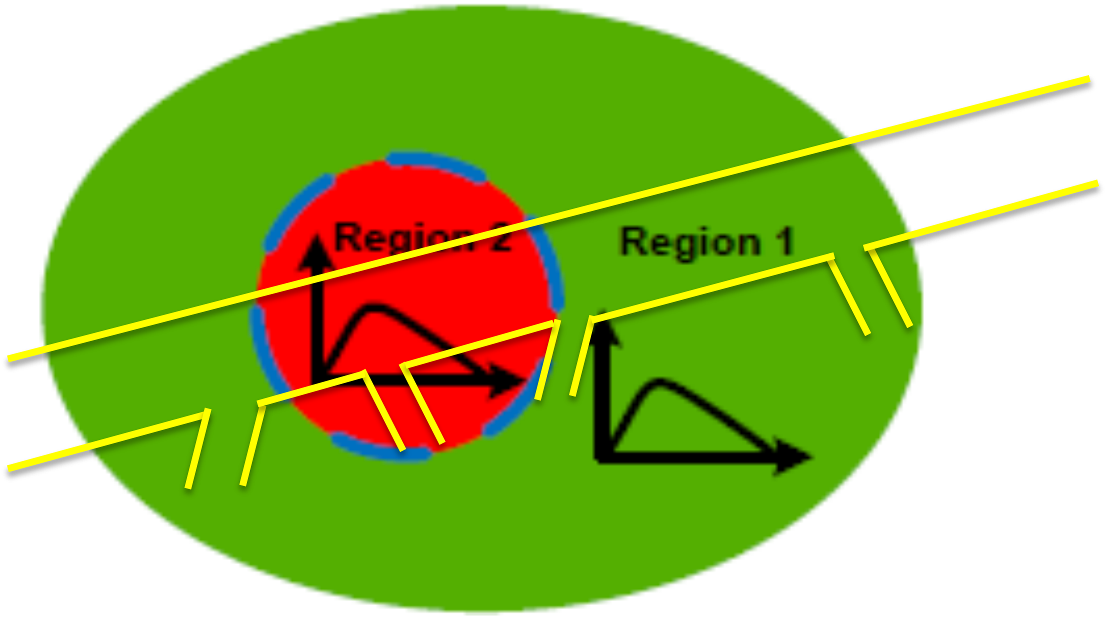


Welcome

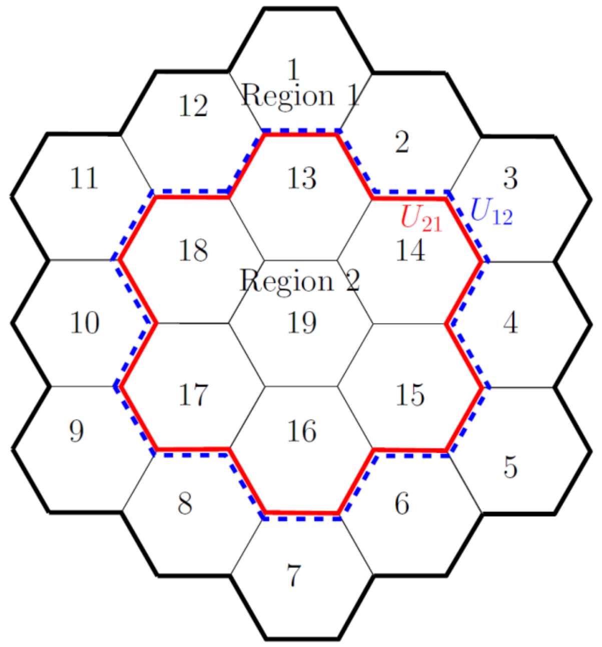
Problem I



Problem II



- Problem III: A multi-region MFD system with heterogeneity and control hierarchy

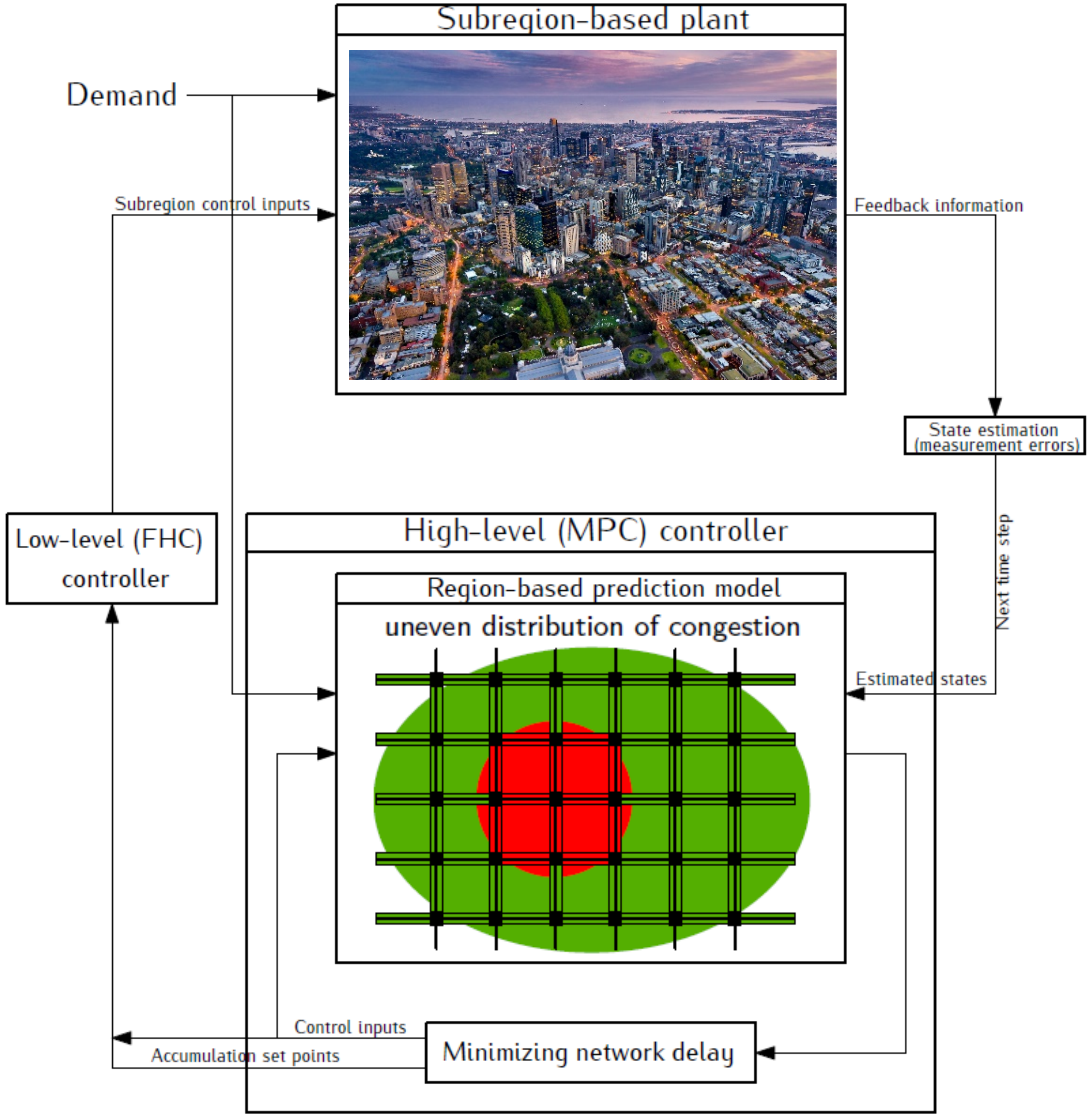


Problem III statement

EPFL

Motivations

- Investigating the effect of heterogeneity on the MFD
- Designing perimeter control strategies to control the heterogeneity
- Hierarchical control (upper and lower level)



Ramezani, Haddad, Geroliminis (2015) –Trans. Res. part B

Subregion-based model

EPFL

$$\frac{\mathrm{d}n_{ii}(t)}{\mathrm{d}t} = q_{ii}(t) - m_{ii}(t) + \sum_{h \in \mathcal{H}_i} u_{hi}(t).\widehat{m}_{hi}^{i}(t)$$

$$\frac{\mathrm{d}n_{ij}(t)}{\mathrm{d}t} = q_{ij}(t) - \sum\nolimits_{h \in \mathcal{H}_i} u_{ih}(t).\widehat{m}_{ij}^{h}(t) + \sum\nolimits_{h \in \mathcal{H}_i; h \neq j} u_{hi}(t).\widehat{m}_{hj}^{i}(t) \qquad \forall\, j \in \mathcal{H}_i$$

$$\frac{\mathrm{d}n_{ir}(t)}{\mathrm{d}t} = q_{ir}(t) - \sum\nolimits_{h \in \mathcal{H}_i} u_{ih}(t).\widehat{m}_{ir}^{h}(t) + \sum\nolimits_{h \in \mathcal{H}_i} u_{hi}(t).\widehat{m}_{hr}^{i}(t) \qquad i \neq r; \forall\, r \notin \mathcal{H}_i$$

$$0 \leq u_{min} \leq u_{ih}(t) \leq u_{max} \leq 1$$

$$m_{ii}(t) = \frac{n_{ii}(t)}{n_i(t)} \times \frac{p_i\big(n_i(t)\big)}{l_i(t)}$$

$$m_{ij}^{h}(t) = \theta_{ij}^{h}(t) \times \frac{n_{ij}(t)}{n_i(t)} \times \frac{p_i\big(n_i(t)\big)}{l_i(t)}$$

$$\widehat{m}_{ij}^{h}(t) = \min(m_{ij}^{h}(t), \frac{m_{ij}^{h}(t)}{\sum_k m_{ik}^{h}(t)} r_{ih}(n_h(t)))$$

$q_{ij}(t)$: demand from subregion *i* to *j* [veh/s]

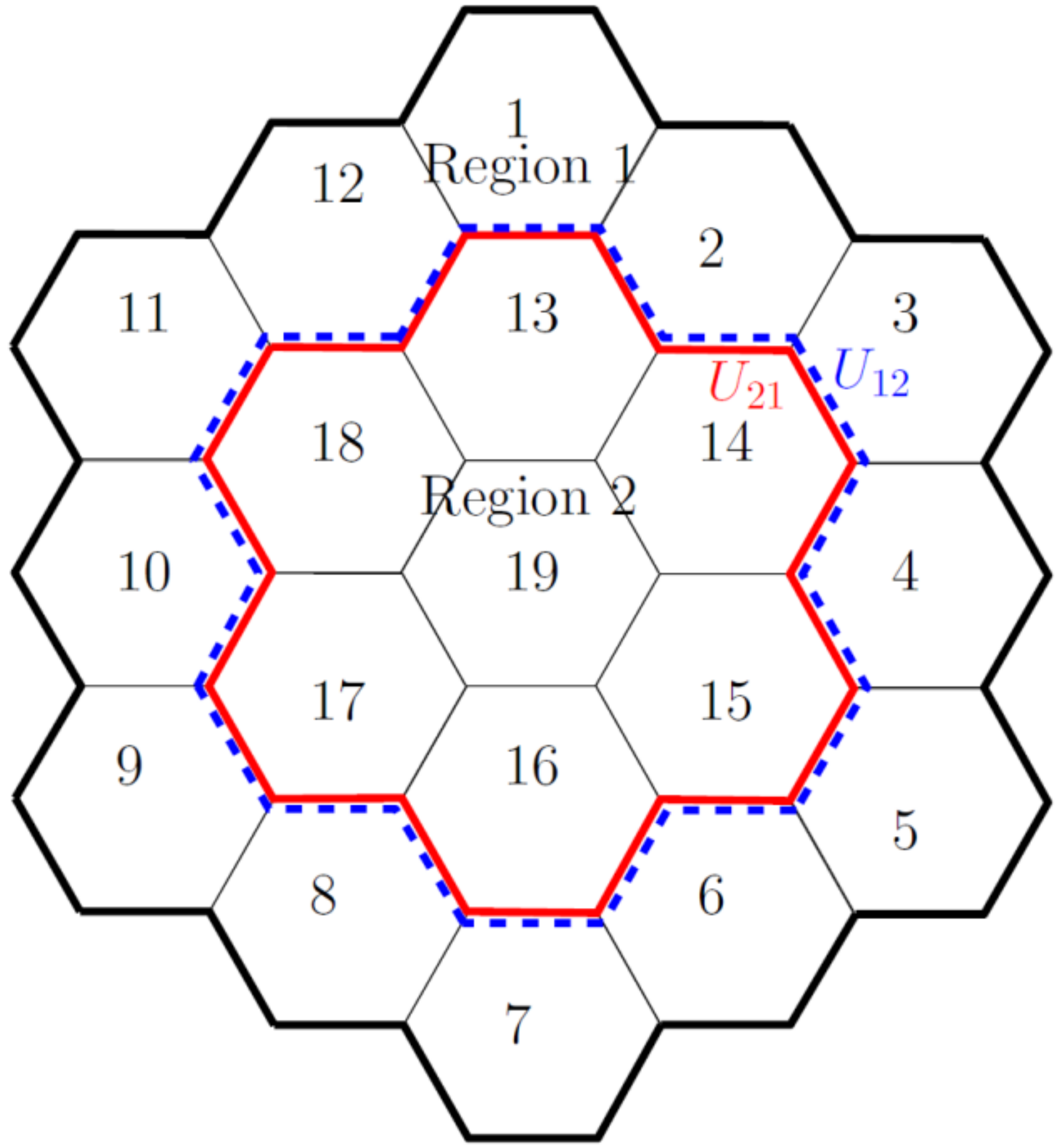
$n_{ij}(t)$: accumulation in subregion *i* with final destination *j* [veh]

$n_i(t)$: accumulation in subregion *i* [veh]

$p_i\big(n_i(t)\big)$: production in subregion *i* [veh.m/s]

$l_i(t)$: average trip length in subregion *i* [m]

$m_{ij}^{h}(t)$: transfer flow from subregion *i* to *h* with final destination *j*

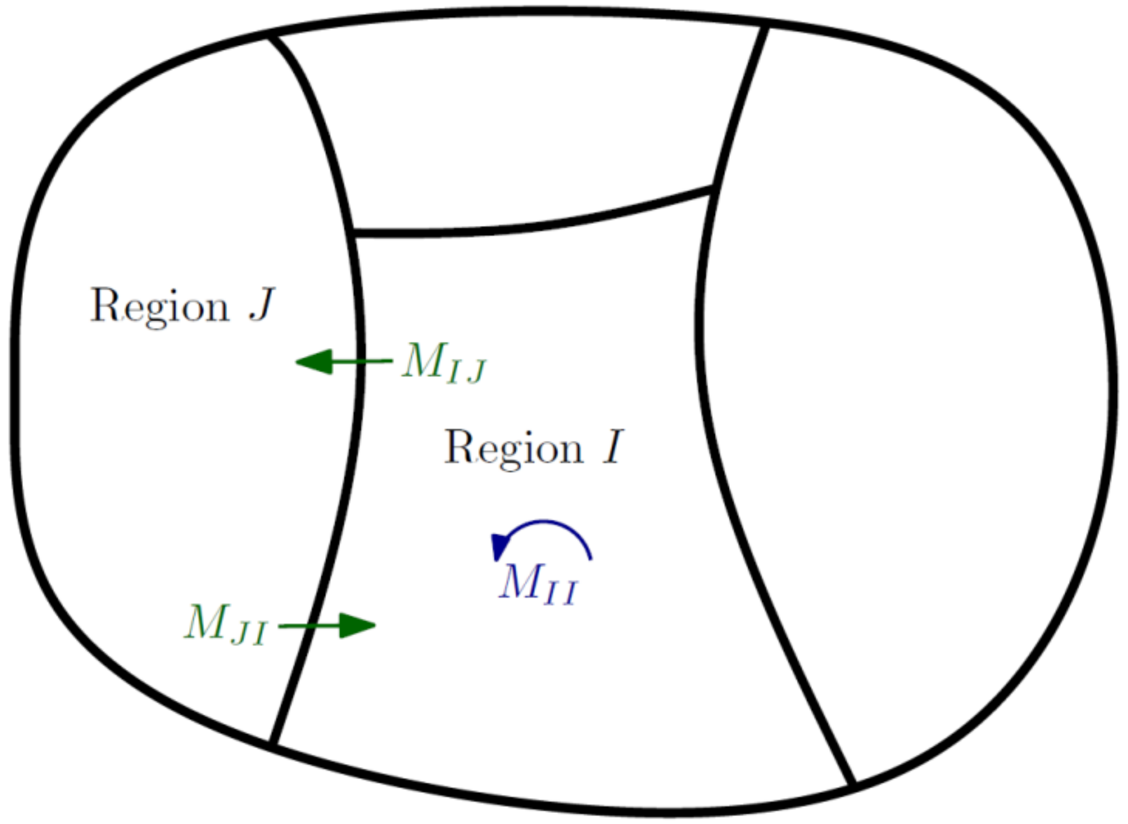


Region-based model

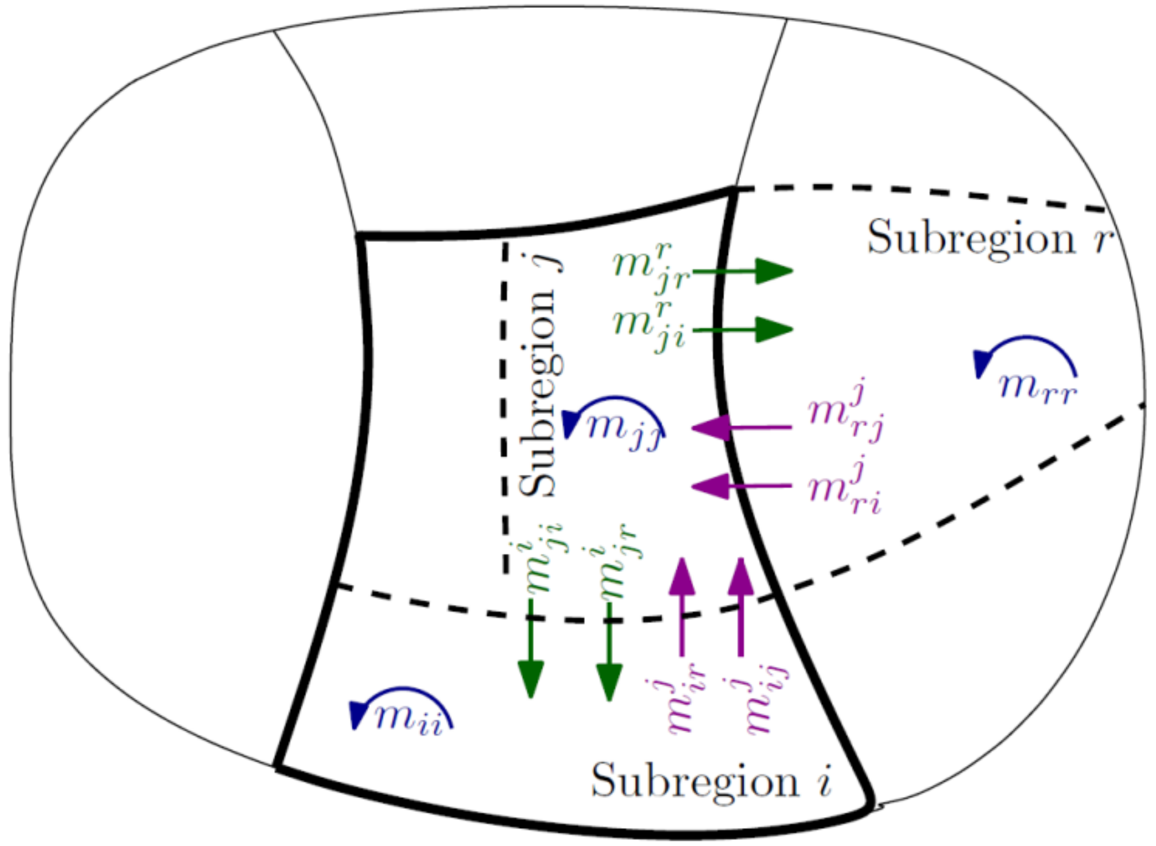
$$\frac{\mathrm{d}N_{II}(t)}{\mathrm{d}t} = Q_{II}(t) - M_{II}(t) + \sum_{J \in \mathcal{H}_I} U_{JI}(t).M_{JI}(t)$$

$$\frac{\mathrm{d}N_{IJ}(t)}{\mathrm{d}t} = Q_{IJ}(t) - \sum_{J \in \mathcal{H}_I} U_{IJ}(t).M_{IJ}(t) \qquad \forall J \in \mathcal{H}_I$$

$$0 \leq U_{min} \leq U_{IJ}(t) \leq U_{max} \leq 1$$



$$\boxed{\begin{aligned} M_{II}(t) &= \frac{N_{II}(t)}{N_I(t)} \times \frac{P_I(N_I(t), \sigma(N_I(t)))}{L_{II}(t)} \\ M_{IJ}(t) &= \frac{N_{IJ}(t)}{N_I(t)} \times \frac{P_I(N_I(t), \sigma(N_I(t)))}{L_{IJ}(t)} \end{aligned}}$$



$Q_{IJ}(t)$: demand from region *I* to *J*; $J \in \mathcal{H}_I$ [veh/s]

$N_{IJ}(t)$: accumulation in region *I* with direct destination *J* [veh]

$N_I(t)$: accumulation in region *I* [veh]

$P_I(N_I(t), \sigma(N_I(t)))$: production in region *I* [veh.m/s]

$L_{II}(t)$: average trip length for trips in region *I* [m]

$L_{IJ}(t)$: average trip length for trips from region *I* to region *J* [m]

Two-level hierarchical control

MPC objective function (High level control):

- $J = \min\limits_{U_{IJ},U_{JI}} \sum_{I\in\mathcal{R}, J\in\mathcal{H}_I} \int_0^{t_f} \left(N_{II}(t) + N_{IJ}(t)\right)\mathrm{d}t$

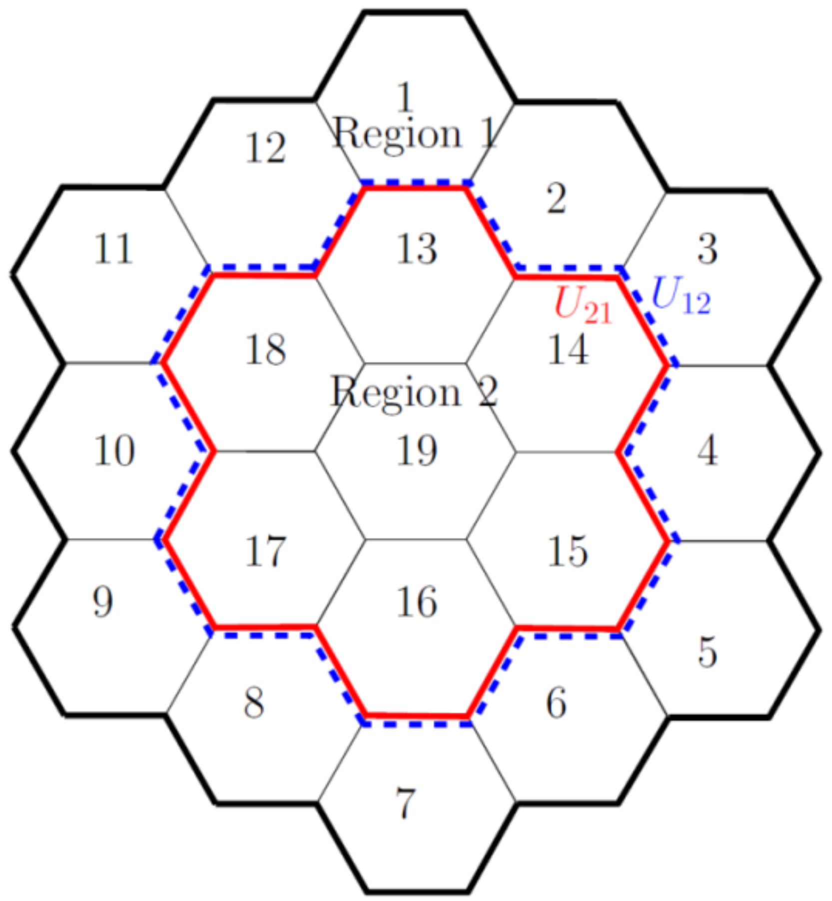
Low-level I controller (feedback homogeneity controller - **FHC**):

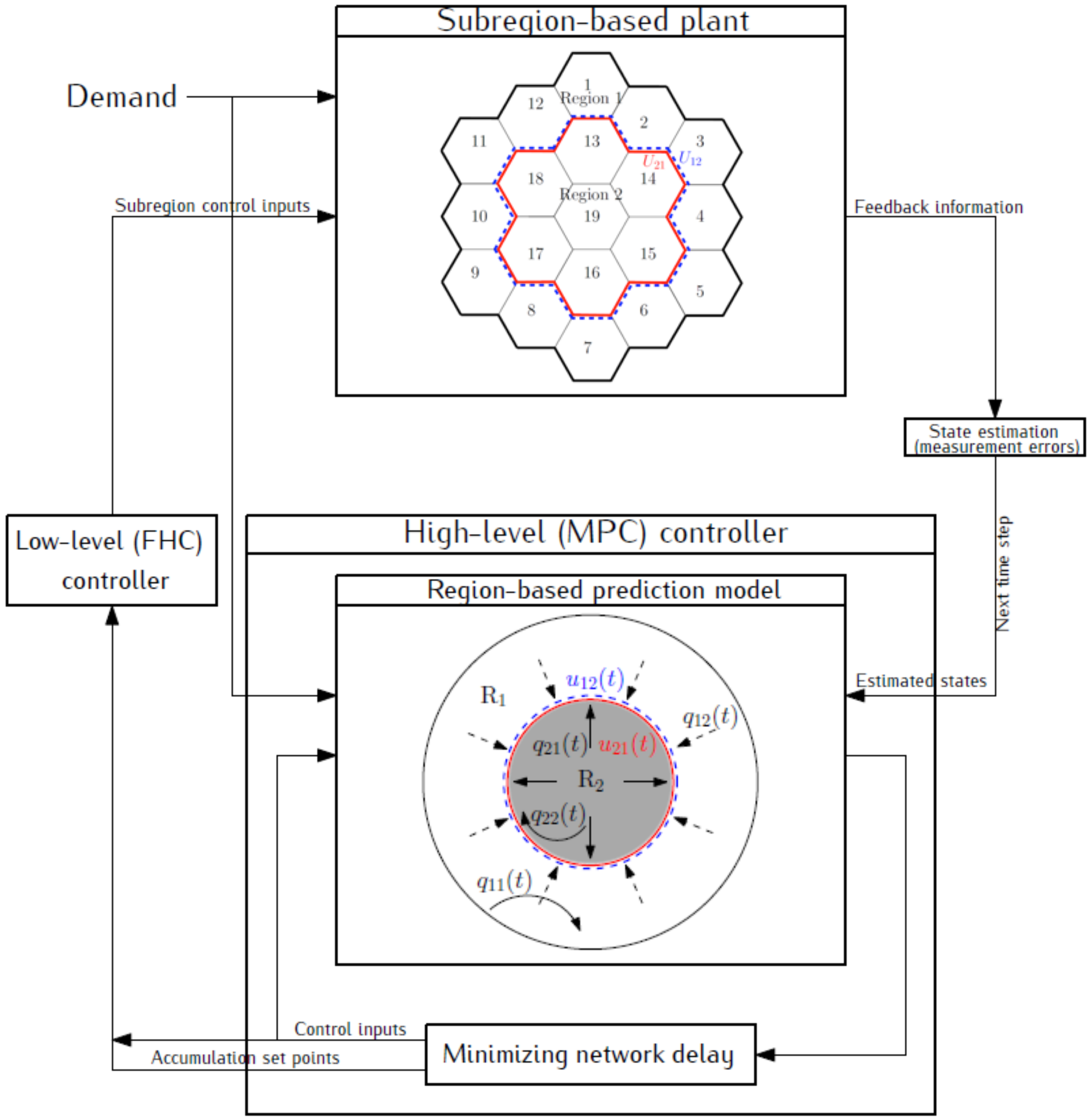
$$u_{ij}(k_c) = u_{ij}(k_c - 1) + K_1 \cdot \left(\frac{N_J(k_c + K_p - 1)}{|\mathcal{SR}_J|} - n_j(k_c)\right) - K_2 \cdot \left(\frac{N_I(k_c + K_p - 1)}{|\mathcal{SR}_I|} - n_i(k_c)\right)$$

subject to:

$$\left|\sum u_{ij} - U_{IJ}\right| < \delta$$

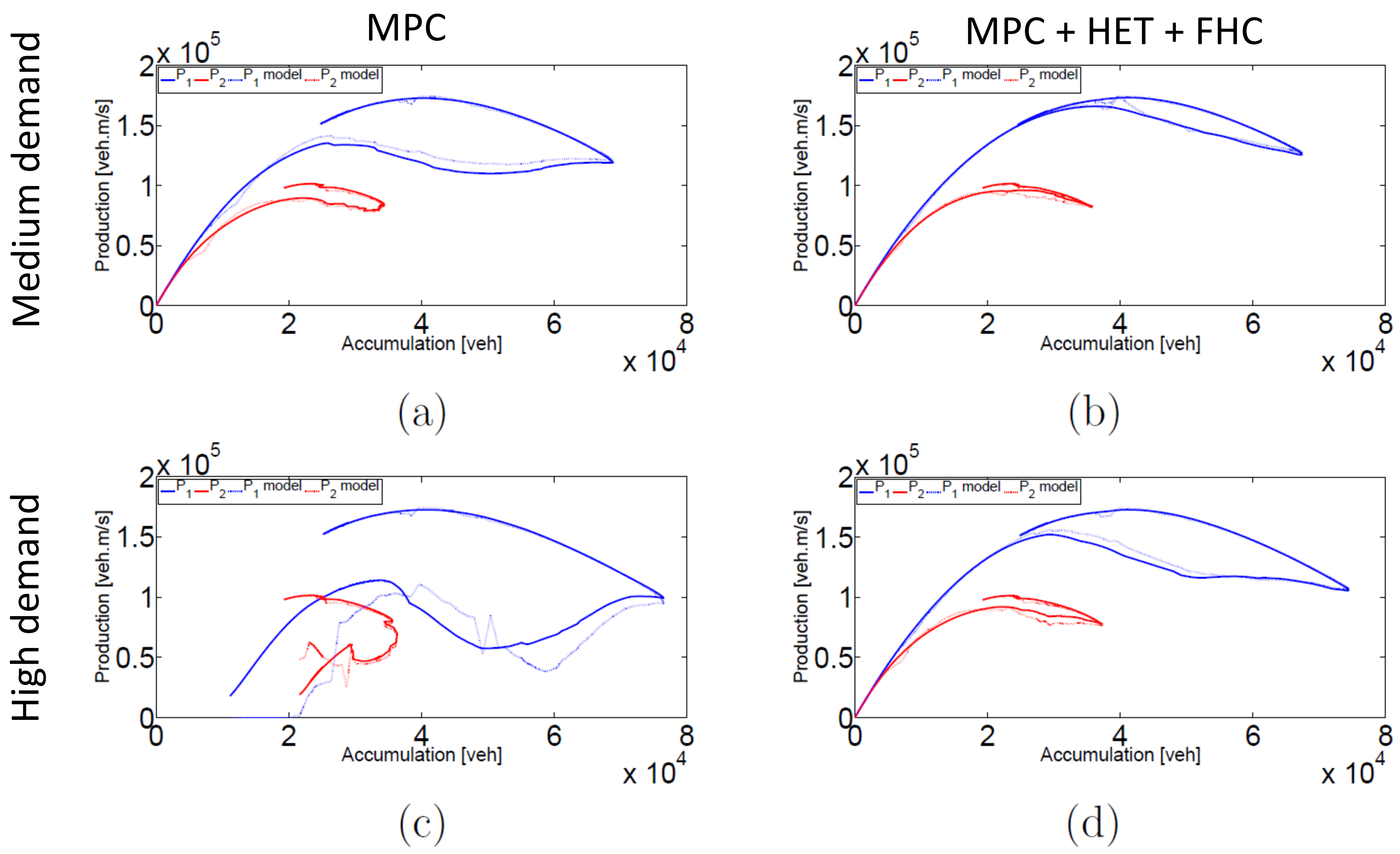
$$0 \leq U_{\min} \leq u_{ij}(k) \leq U_{\max} \leq 1$$





K_p: MPC prediction horizon

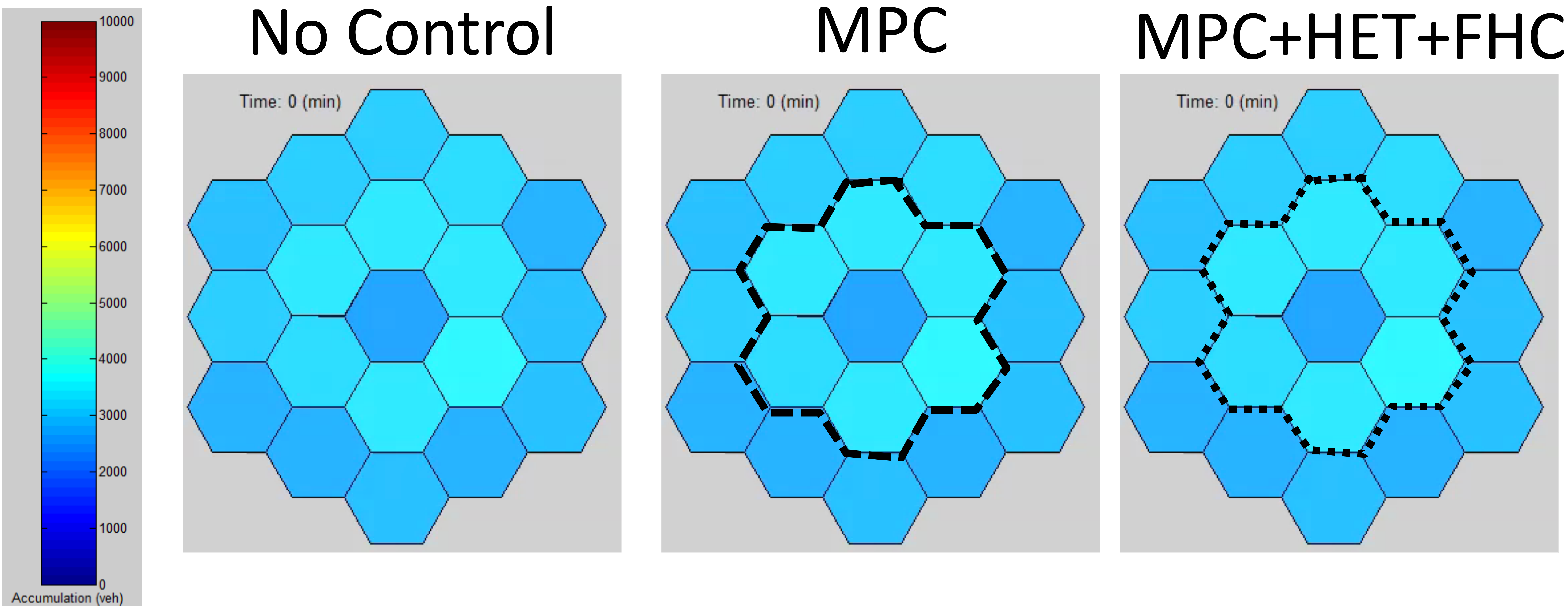
Performance of different controllers



Total network delay (veh.sec × 10^6)

Demand	No Control	MPC	MPC + HET + FHC
Medium	1069.4	573.8	518.0
High	1204.4	930.5	636.8

An illustration of different controllers



MFD control - Summary